Laboratory: Resonance

OBJECTIVES:

After successfully completing this laboratory, you should be able to:

- Determine the relation between the frequencies of resonant vibrations of a wire with a given tension, length, and mass per unit length.
- Determine the relation between the velocity of waves on a wire and the tension and mass per unit length of the wire.
- Understand the relevance of resonant frequencies of wires used in particle physics experiments.

EQUIPMENT:

- Function Generator
- 2 alligator clips
- Ruler
- Large magnet
- Lab stands
- Wires with m/L of $\approx$0.00014 kg/m
- Pulley, rods, weight set
- 2 Table clamps
- 2 Banana leads

Physics Background:

The relation between frequency, f, speed of propagation, v, and wavelength, λ, for a wave is given by

$$f = \frac{v}{\lambda} \tag{1}$$

Waves in wires will travel at a velocity that is dependent on the tension, T, of the wire and the mass per unit length ($\mu \equiv m/L$) of the wire. When a wire under tension is pulled sideways and released, the tension in the wire is responsible for accelerating a particular segment back toward its equilibrium position. The acceleration and wave velocity increase with increasing tension in the wire. Likewise, the wave velocity, v, is inversely related to the mass per unit length of the wire. This is because it is more difficult to accelerate (and impart a large wave velocity) to a massive wire compared to a light wire. The relation between the wave velocity, tension, and mass per unit length is given by

$$v = \sqrt{\frac{T}{\mu}} \tag{2}$$

If the wire is fixed at both ends, traveling waves will reflect from the fixed ends, creating waves traveling in both directions. The waves traveling in opposing direction will form a superposition. Depending on the frequency of these waves, they can reinforce each other or cancel each other out. As an example, when the string is vibrated at exactly the right frequency, a crest moving toward one end and a reflected trough will meet at some point along the string. The two waves will cancel at this point, this is called a node. The resulting pattern on the string is one in which the wave appears to stand still, and we have what is called a standing wave on the string. At the nodes, there is no motion in the string. The points which vibrate with maximum amplitude are called anti-nodes. If the string has length L and is fixed at both ends, the condition for achieving standing waves is that the length of the string be equal to a half-integral number of wavelengths.

$$L = n\frac{\lambda_n}{2} \quad \text{or} \quad \frac{1}{\lambda_n} = \frac{n}{2L} \tag{3}$$

Substituting Equations 2 and 3 into Equation 1 gives

$$f_n = \frac{v}{\lambda_n} = \frac{n}{2L}\sqrt{\frac{T}{\mu}} \tag{4}$$

Applications in High Energy Physics:

The interplay of resonance behavior with tension and frequency is one of the major considerations taken into account when designing detectors for high energy physics experiments.

Tracking Chambers

Wire chambers (or “drift” chambers) are commonly used in particle physics and provide high-resolution pictures of particle interactions, and can be read out electronically at a rate fast enough to keep up with the accelerator sources used today. Original development was done by Georges Charpak in the late 1960s, for which he eventually won the Nobel Prize for in 1992. These detectors may contain tens of thousands of wires, installed with precise orientations and positions. Detecting an electrical signal on these wires indicates a charged particle travelled near the wire, and the position of all the wires with signals can be used to reconstruct the path of the traveling particle.

The position of the wires within the tracking chamber must be precisely controlled, and the wires must not sag too much or they will degrade the detector's capabilities. Controlling the tension of the wires is crucial. These detectors are often located in the presence of strong magnetic fields, so studies must be done before construction to ensure the wires will not be subjected to any resonant forces that might impact the detector performance.

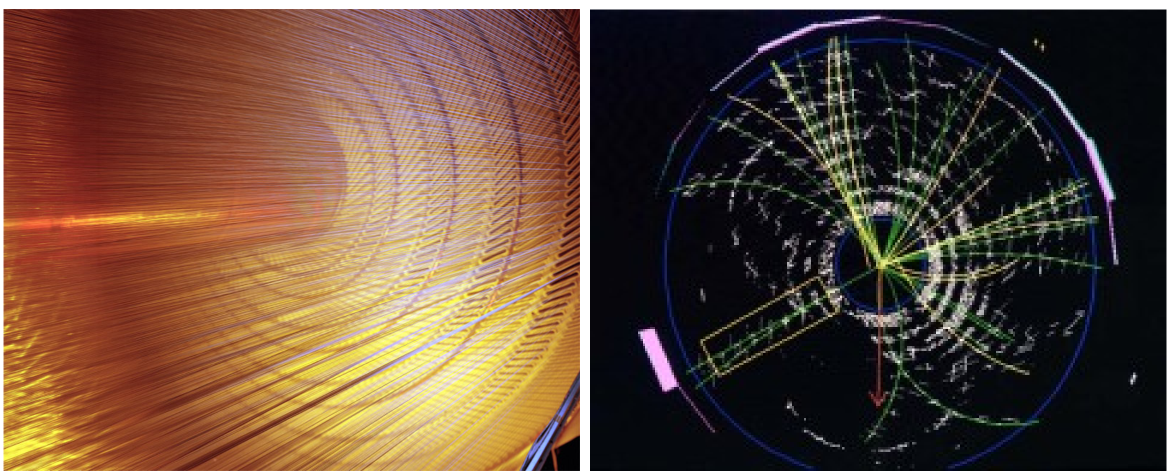

Figure 1: Left: Wire chamber. Right: Tracks detected in a wire chamber.

Time Projection Chambers (TPCs)

Time Projection Chambers are a variation on Charpak's wire chambers. In these detectors ionization, created by the passing of a charged particle, is drifted over significant distances before ultimately encountering wires that develop electrical signals when the ionization approaches. As with the traditional wire chambers, the position of wires that have detected signals can be used to reconstruct the path of the traveling particle. Unlike traditional wire chambers, the true position of the original particle needs to be inferred based on the known drift velocity of ionization through the TPC, and the amount of time that elapsed between creation of the ionization and its arrival at a wire. Multiplying drift velocity and drift time gives the true coordinates of the instigating interaction.

TPCs immersed in liquid argon (temperature of 87 K) are incredibly powerful for studying neutrino interactions. Wires immersed in liquid argon will develop considerable tension due to thermal contraction. Careful design must be followed to allow for this thermal contraction without exceeding the breaking point of the chosen wire material. Resonant frequency measurements are used to precisely determine the tension of the TPC wires prior to insertion in the liquid argon.

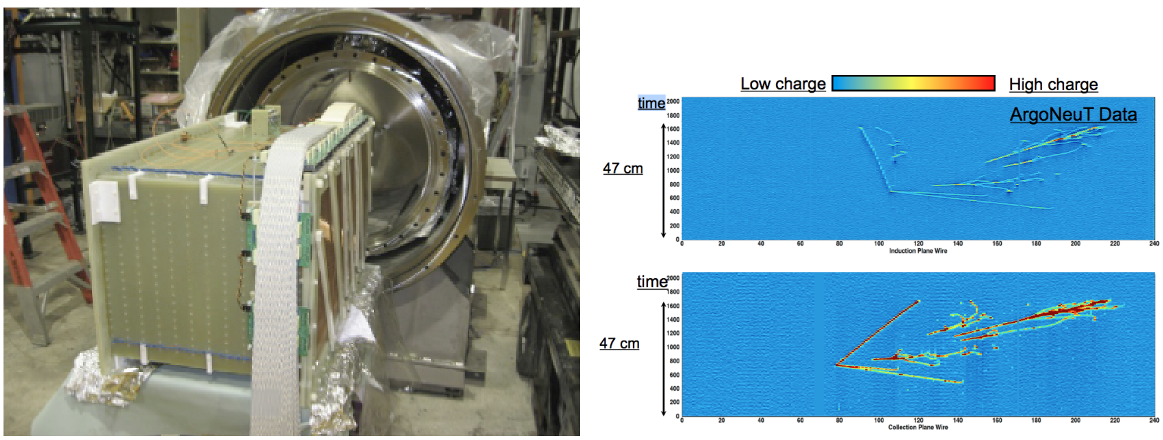

Figure 2: Left: TPC Detector and Cryostat. Right: Tracks in TPC Detector.

Silicon detectors

Silicon detectors are commonly used for precision particle tracking. Unlike tracking chambers and TPCs, the signal of interest develops in silicon semiconductor chips, and not in wires. However, they typically employ very short (∼2mm) length wires that are bonded to the silicon chips and then used to deliver data and power. These wires are located in a non-zero magnetic field present inside the detectors.

On the CDF silicon detector at Fermilab, resonance frequencies of ≈15kHz were observed to cause wire bonds to vibrate, and eventually break under increased strain, which reduced the capability of the detector. New protection mechanisms were developed to detect resonance conditions developing in these wire bonds, and to halt them before they caused damages.

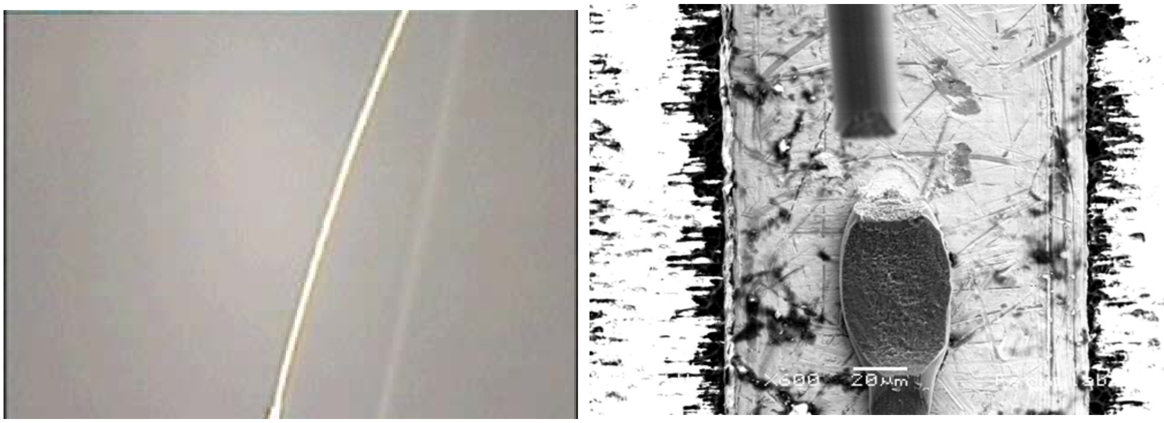

Figure 3: Left: CDF wire bond vibrating. Right: Broken CDF wire bond.

EXPERIMENTAL SETUP:

1. Secure lab stands to opposite ends of a long table with C-clamps. Attach pulley to one of the lab stands.

2. Cut a length of wire, with mass per unit length of $\approx$0.00014 kg/m, that spans between the two clamps plus an additional length to hang down over the pulley. A length between 1m and 2 m is a good starting point.

3. Attach one end of the wire to the clamp without the pulley, and support a mass (of between 50 g and 100 g) from the other end that hangs over the pulley.

4. Connect banana leads from function generator to each end of the wire using alligator clips.

5. Position magnets near the center of the strung wire, making sure to not touch the wire with the magnet. **CAUTION: The magnets used in this activity are heavy, and if brought close together in the correct orientation they will experience a significant attraction which could lead to pinched fingers**.

6. Turn on the function generator, making sure it is set to produce a sine wave with an amplitude of at least 3 Volts.

PART 1: Relationship between Resonant Frequencies and the Number of Nodes

1. Place the magnet near the center of the wire.
2. Slowly increase function generator frequency from 0 Hz until the wire vibrates resonantly, with one antinode. Record the frequency and the number of antinodes.
3. Move the magnet to where an antinode will be next, and increase the frequency until there are two antinodes. Record the frequency and the number of antinodes.
4. Repeat until you are no longer able to discern the next expected resonant frequency.

Analysis:

1. Plot the values of f_n and n that you recorded, with n values on the X axis and f_n values on the Y axis.
2. The slope of the graph corresponds to the measured value of the fundamental frequency, f_1. Determine the slope.
3. The y-intercept of a straight line crossing through the data points should be zero. The actual value of this y-intercept gives a measure of the uncertainty in the measured value of the fundamental frequency.
4. How does the measured value of the fundamental frequency, f_1, compare with the calculated value using Equation 4?
5. Calculate the wave velocity, v, using Equation 2 and also using Equation 1. How do the two results compare?

PART 2: Relationship between Resonant Frequencies and Tension

1. Using the same wire from part 1, hang a 100 g weight from the wire.
2. Vary the frequency up through at least the third (n=3) resonance, recording the frequency and number of antinodes.
3. Repeat the previous step with four more tension values, ranging from 1 N (for the 100 g weight) through 15 N (for a 1500 g weight). Record the values for mass/tension, along with the resonant frequencies and number of antinodes.

Analysis:

1. For each value of tension used, plot the values of f_n and n that you recorded, with n values on the X axis and f_n values on the Y axis.
2. As in Part 1, the slope of each of these lines corresponds to the fundamental frequency for the wire under a given tension. Determine the slope for each value of tension recorded.
3. Calculate the wave velocity for each value of tension used, again using both Equation 2 and Equation 1.
4. How does the wave velocity change with increasing tension?

PART 3: Relationship between Resonant Frequency and Wire Length

1. Prepare four additional wires, of the same mass per unit length as the wire used in Part 1 and 2, each with increasing length. Example length could be 1.5m, 2.0m, 2.5m, 3.0m.

2. For each wire prepared, repeat the steps in Part 1. Use the same tensioning mass that was used for Part 1.

3. Record the resonant frequencies and number of antinodes, along with the length of wire.

Analysis:

1. For each value of wire length L used, plot the values of f_n and n that you recorded, with n values on the X axis and f_n values on the Y axis.

2. As in Part 1, the slope of each of these lines corresponds to the fundamental frequency for the wire of length L. Determine the slope for each value of L used.

3. Calculate the wave velocity for each value of length used, again using both Equation 2 and Equation 1.

4. Does the wave velocity change with increasing wire length?

Further Investigation:

1. Describe an experiment for determining the relationship between resonant frequency and mass per unit length of wire.

2. Describe an experiment for determining the relationship between resonant frequency and the temperature of the wire. Consider what would happen to the wire if it was suddenly cooled to a very low (*e.g.* Liquid Nitrogen) temperature, or warmed to a very hot temperature. Such an experiment can be thought of as building a thermometer.

3. Based on what you've discovered in this exercise, explain the importance of "tuning" guitars (or other stringed instruments). Describe what is happening in this "tuning" process.

4. Explain why the wire vibrates in the orientation it does (*e.g.* - side to side, or up and down) relative to the magnet orientation. Hint: Consider the force on the wire due to the magnet in your answer.

5. Aim a strobe light with tunable frequency at a wire under resonance. See if you can "freeze" the wire in place.