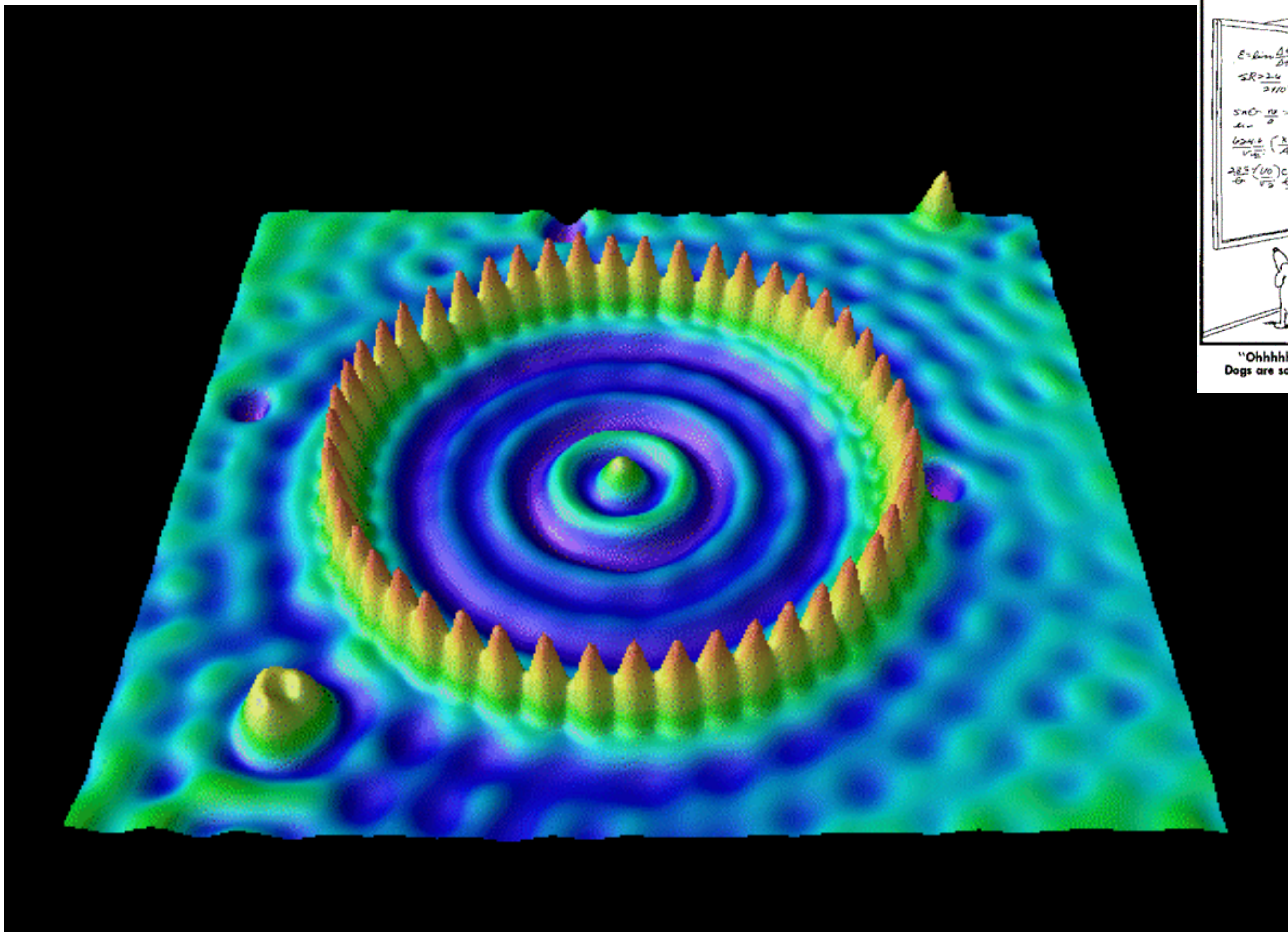
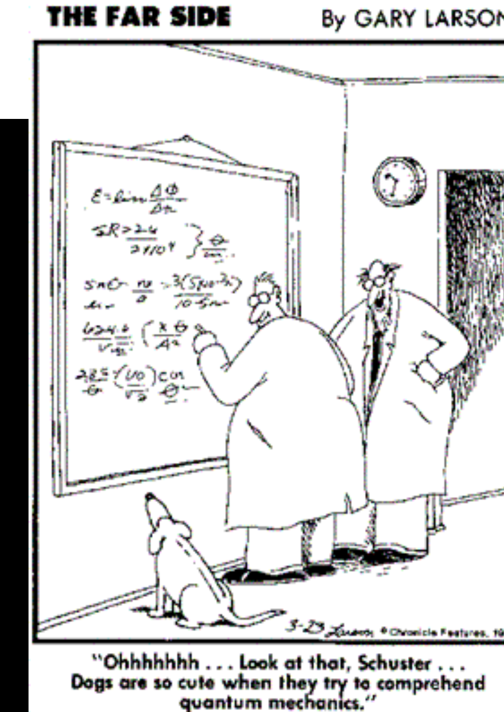
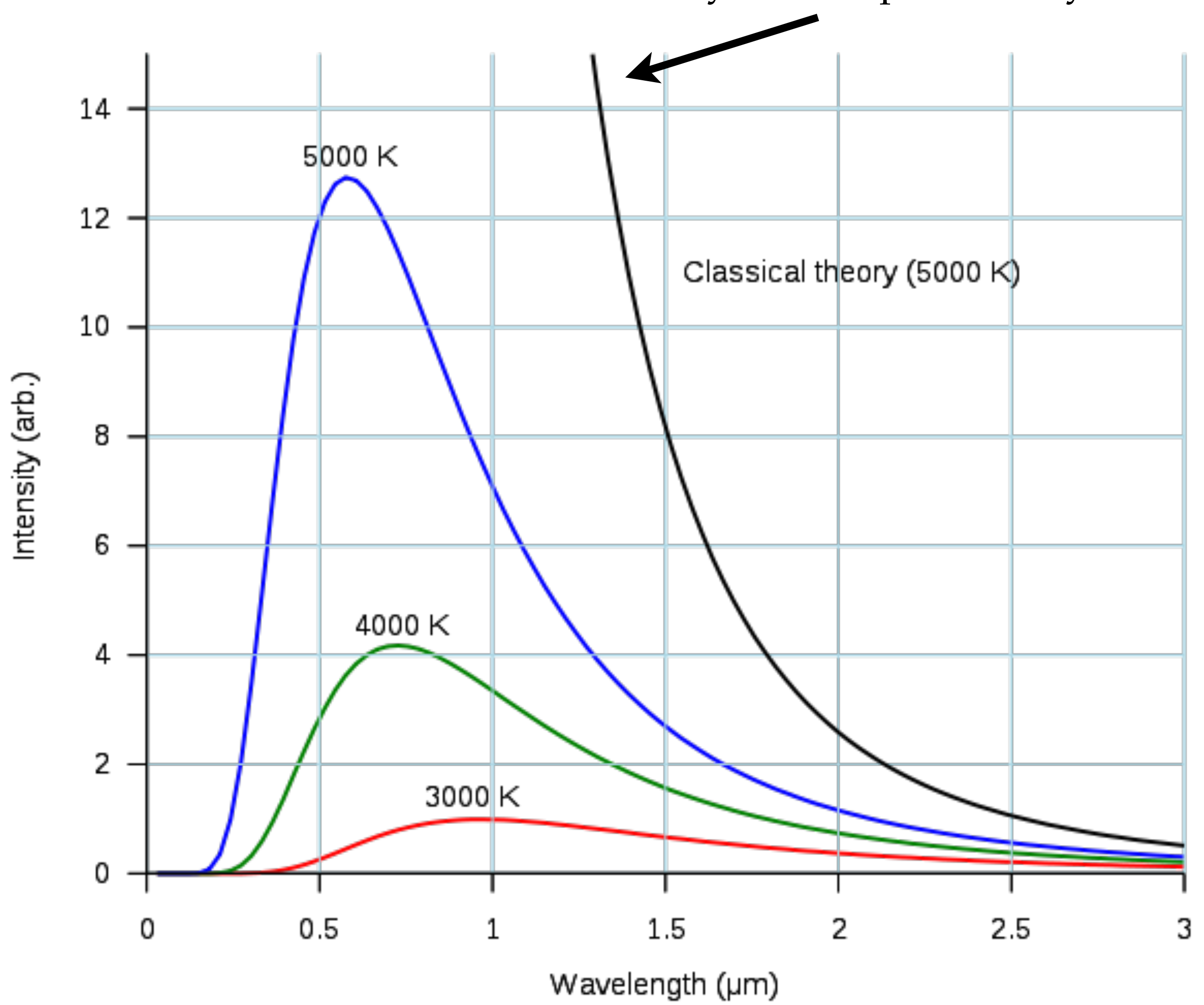


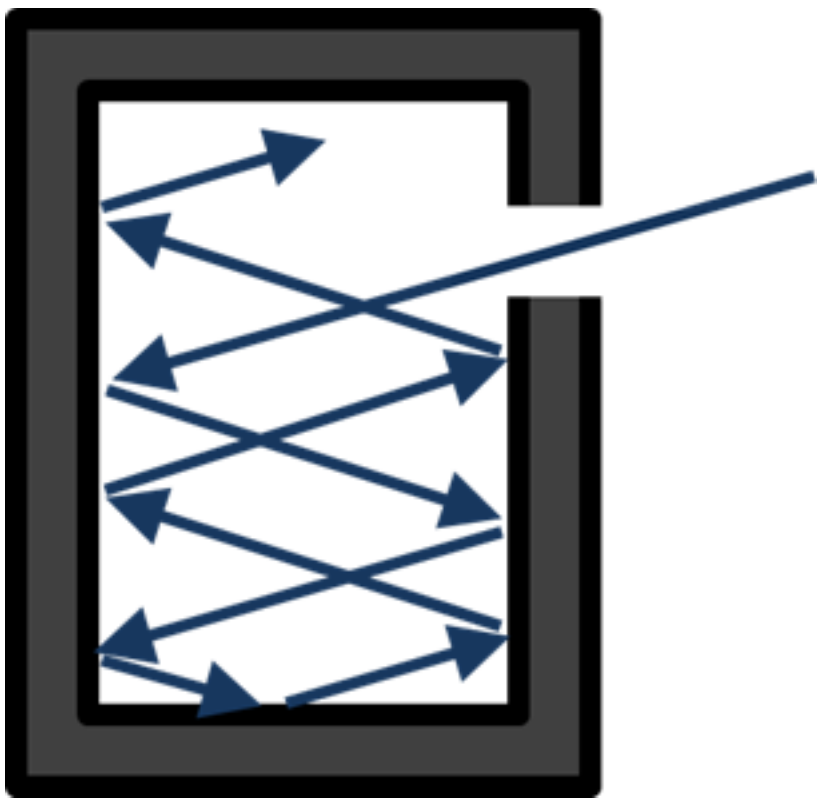


Quantum Mechanics:
QuarkNet Lecture

“Blackbody” = Idealized physical body that absorbs all incident electromagnetic radiation.

Classical theory blows up to infinity at low wavelength.





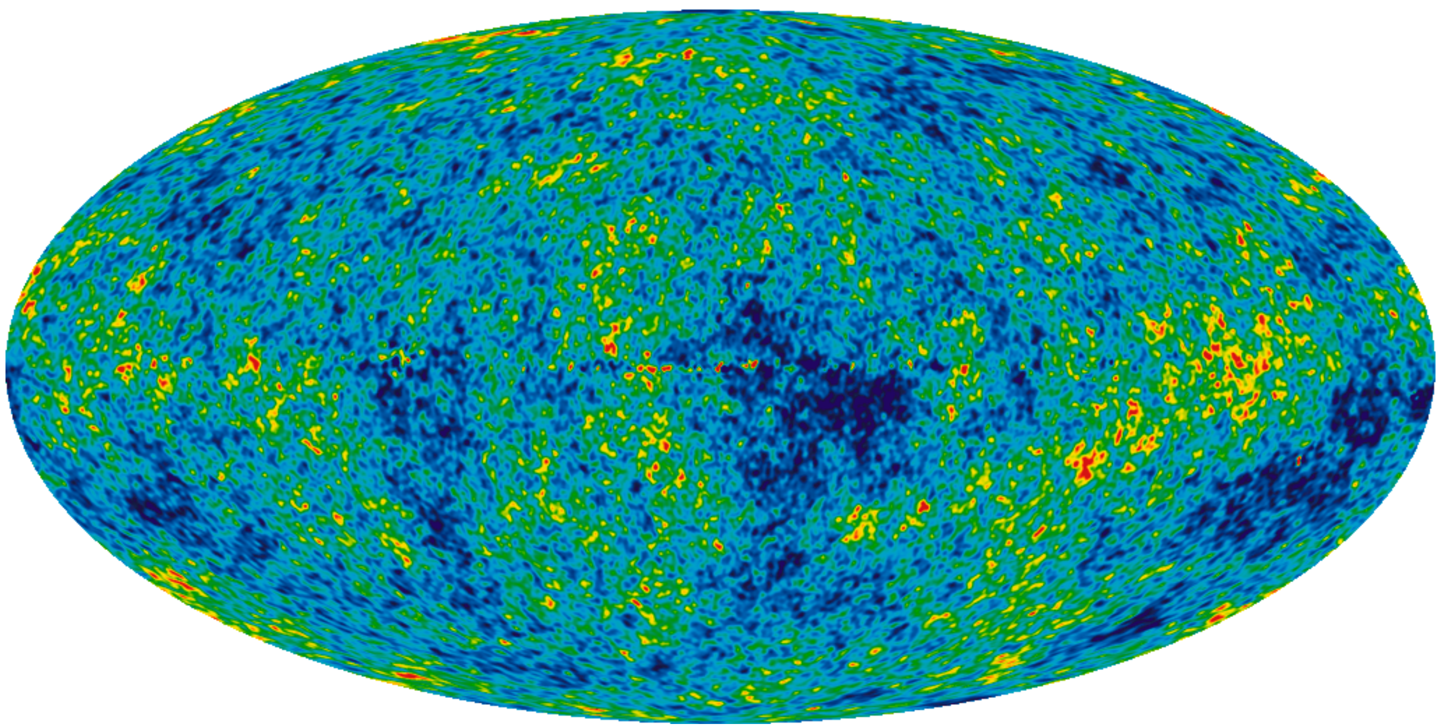
Blackbody cavity

Early theoretical models to explain Blackbody spectrum had divergence at low wavelength: “ultraviolet catastrophe”.

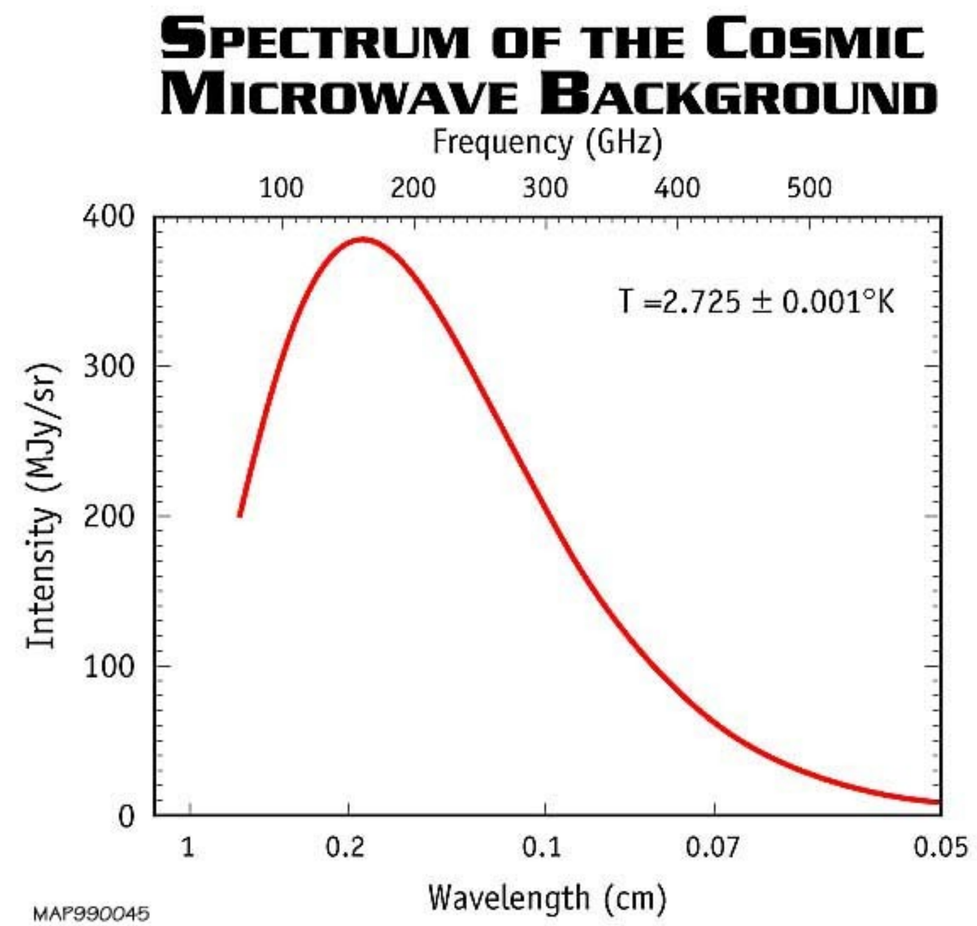
Max Planck developed model of Blackbody spectrum in terms of "quantum oscillators" emitting all frequencies. Formula for spectrum contained constant, *h*, that he tuned to fit Blackbody data.

Planck's Constant

$$h = 6.626\ 069\ 57(29) \times 10^{-34}\,\mathrm{J \cdot s}$$



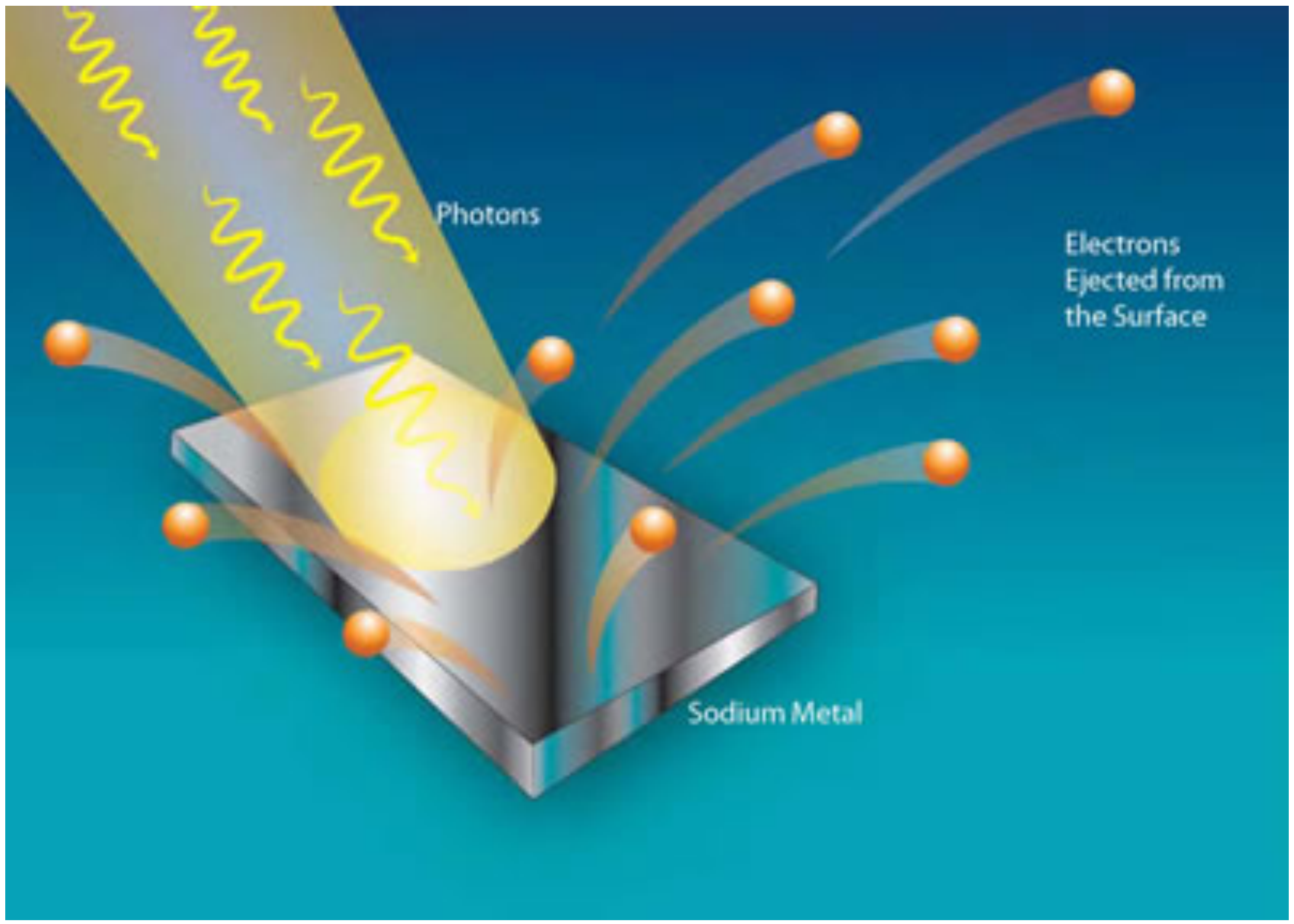
T = 2.725 K



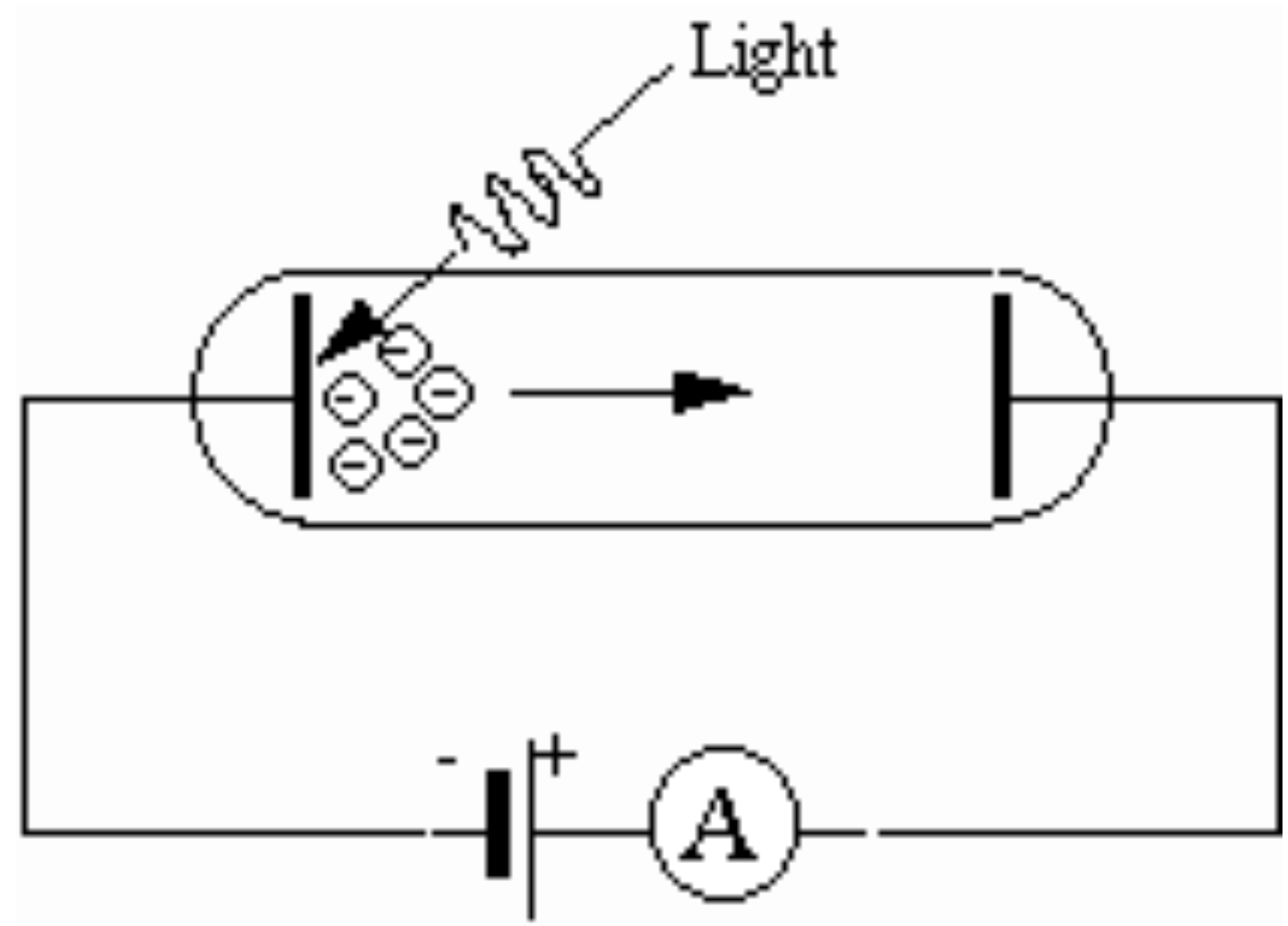
$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/(\lambda k_\mathrm{B}T)} - 1}$$

Aitoff Projection

Photoelectric Effect



Shining light on a metal emits electrons.

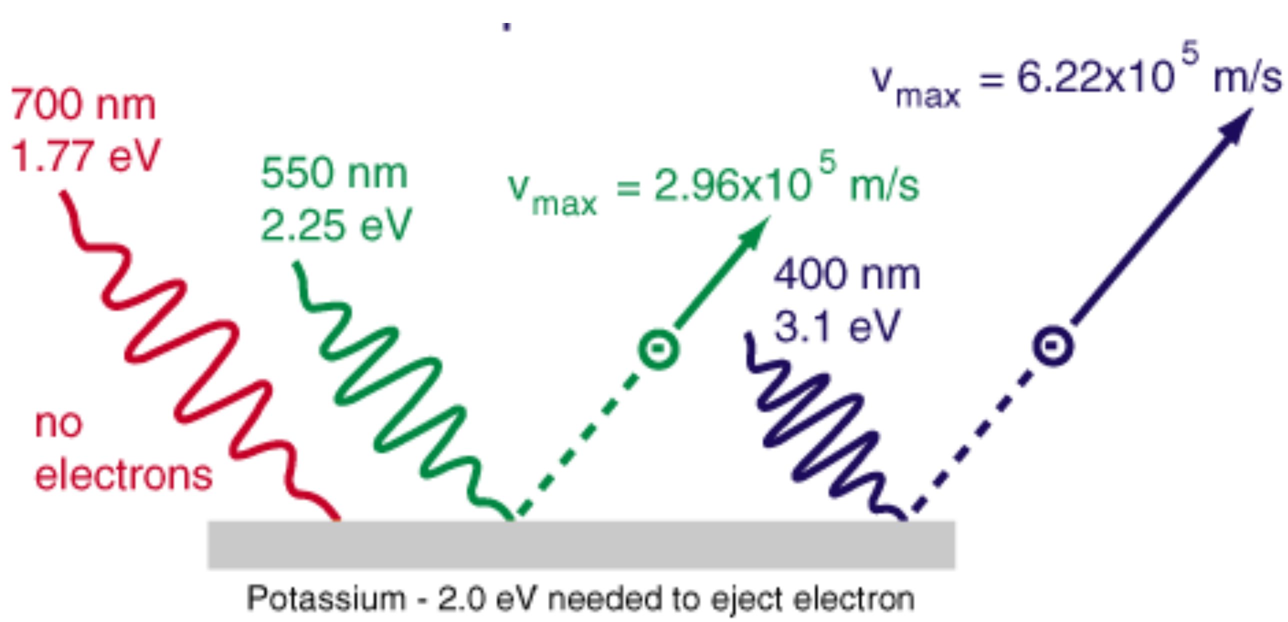


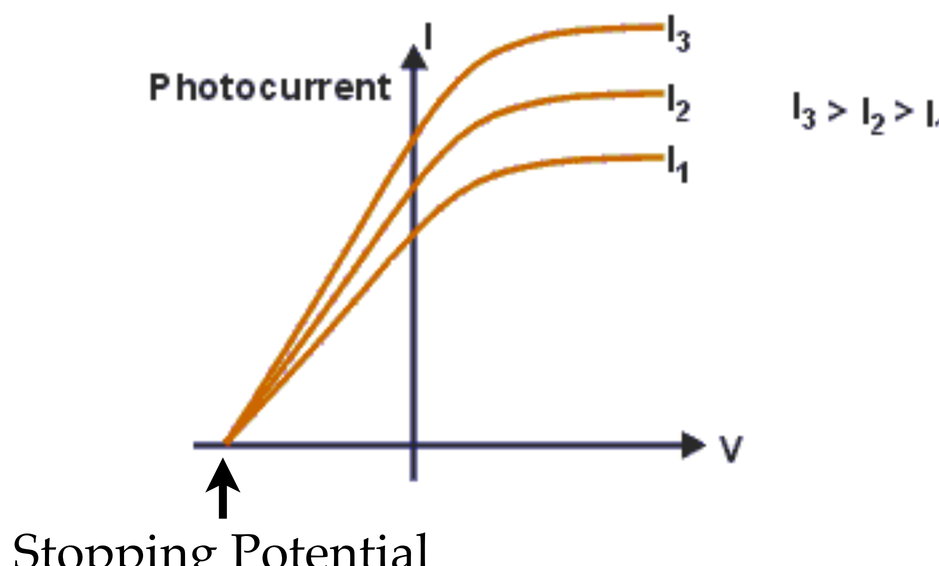
- In a circuit, we can detect a current created when we shine light on a “photocathode”.
- We can adjust Voltage to increase/ decrease the “photocurrent”.

Photoelectric Effect

$$E_{\text{photon}} = h\nu$$

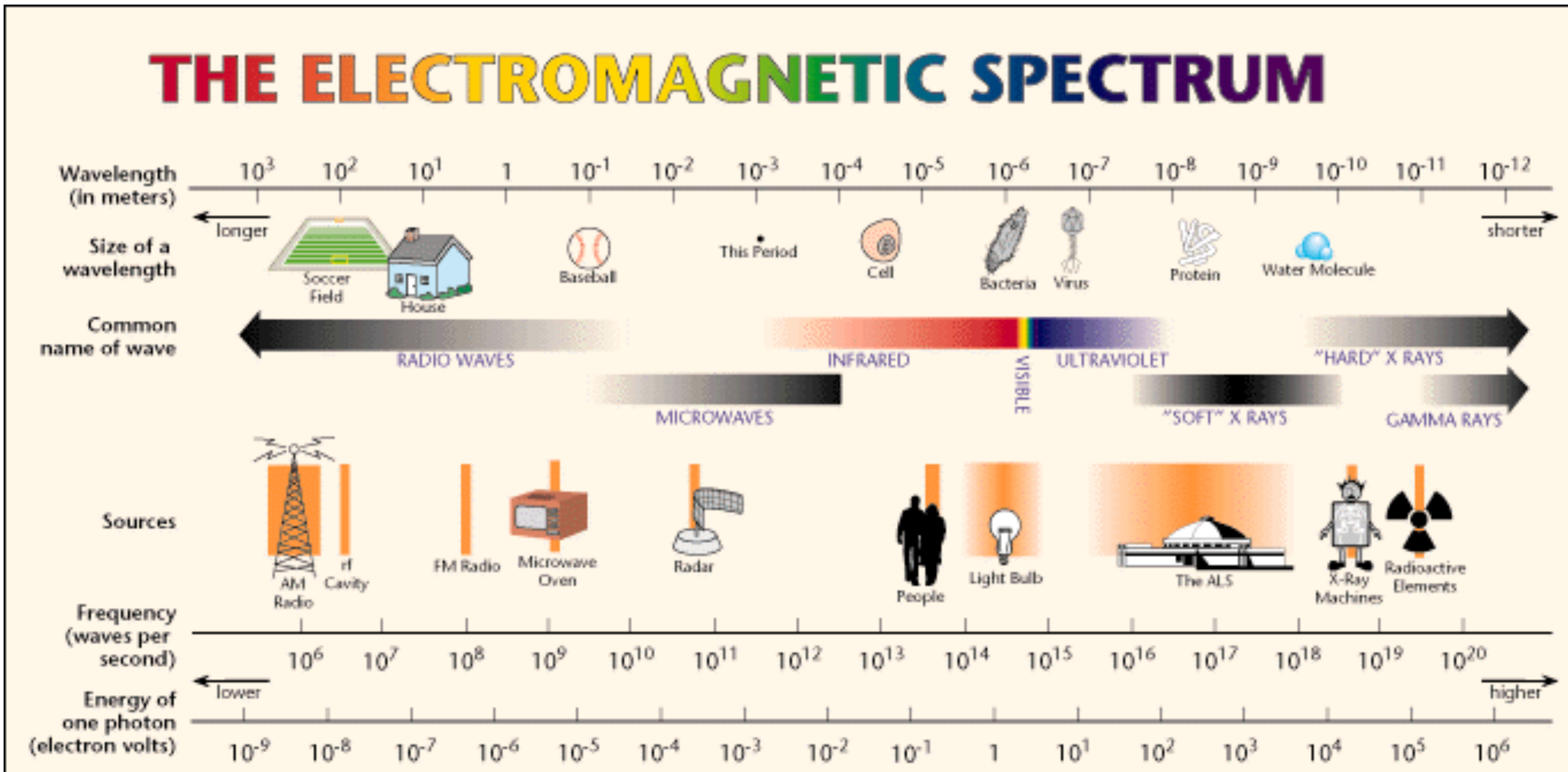
- Einstein (1905) suggests light is composed of particles, called photons, each with energy proportional (by Planck's constant) to its frequency, ν.
- If photon energy is greater than the "work function" of the cathode metal, a photoelectron can be ejected.
- If photon energy is less than the "work function" of the cathode metal, no photoelectron will be ejected, no matter how many photons hit the cathode.



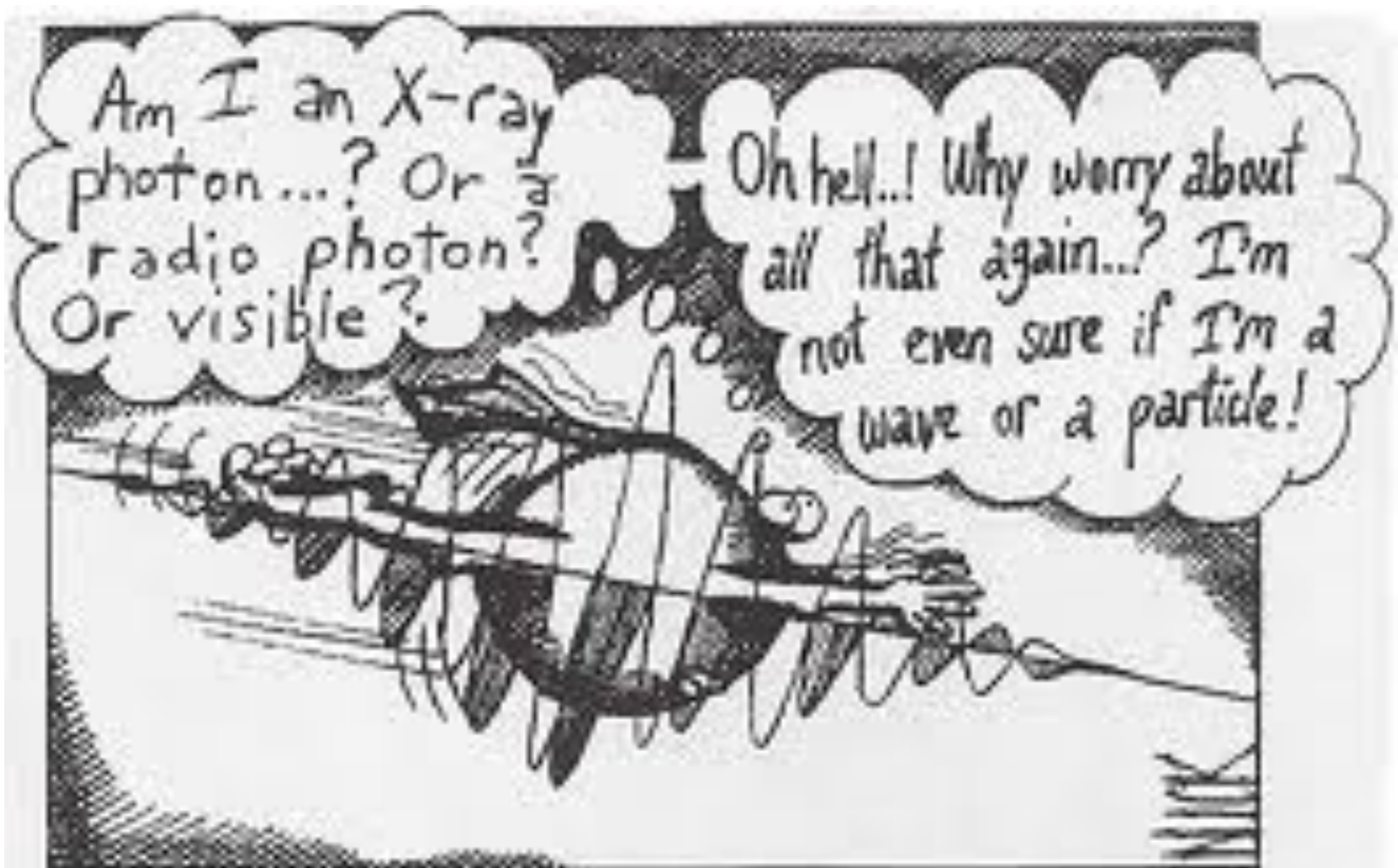


Light is a Particle

Photoelectric effect shows particle nature of light, and contradicts wave nature, since increasing intensity won't liberate photoelectrons unless the frequency is above threshold.



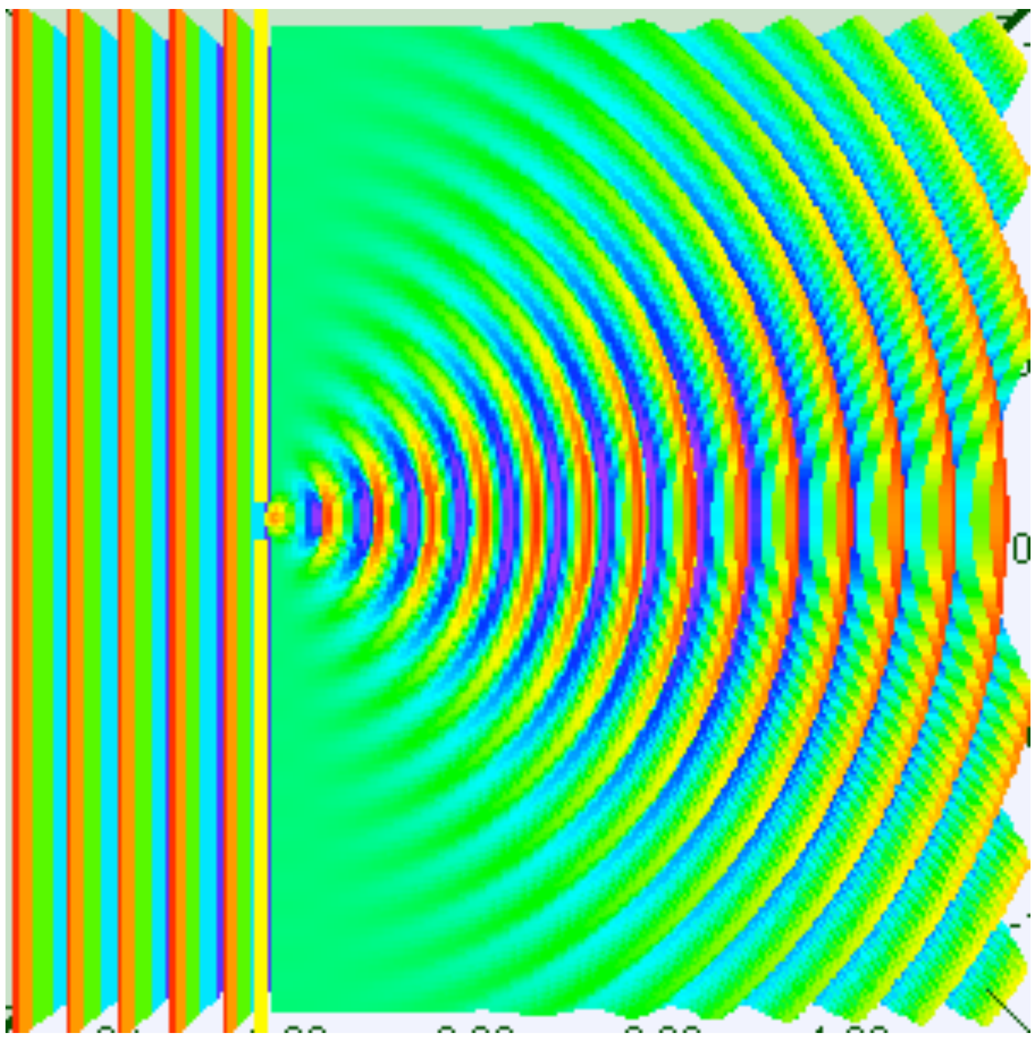
LIGHT IS A PARTICLE!



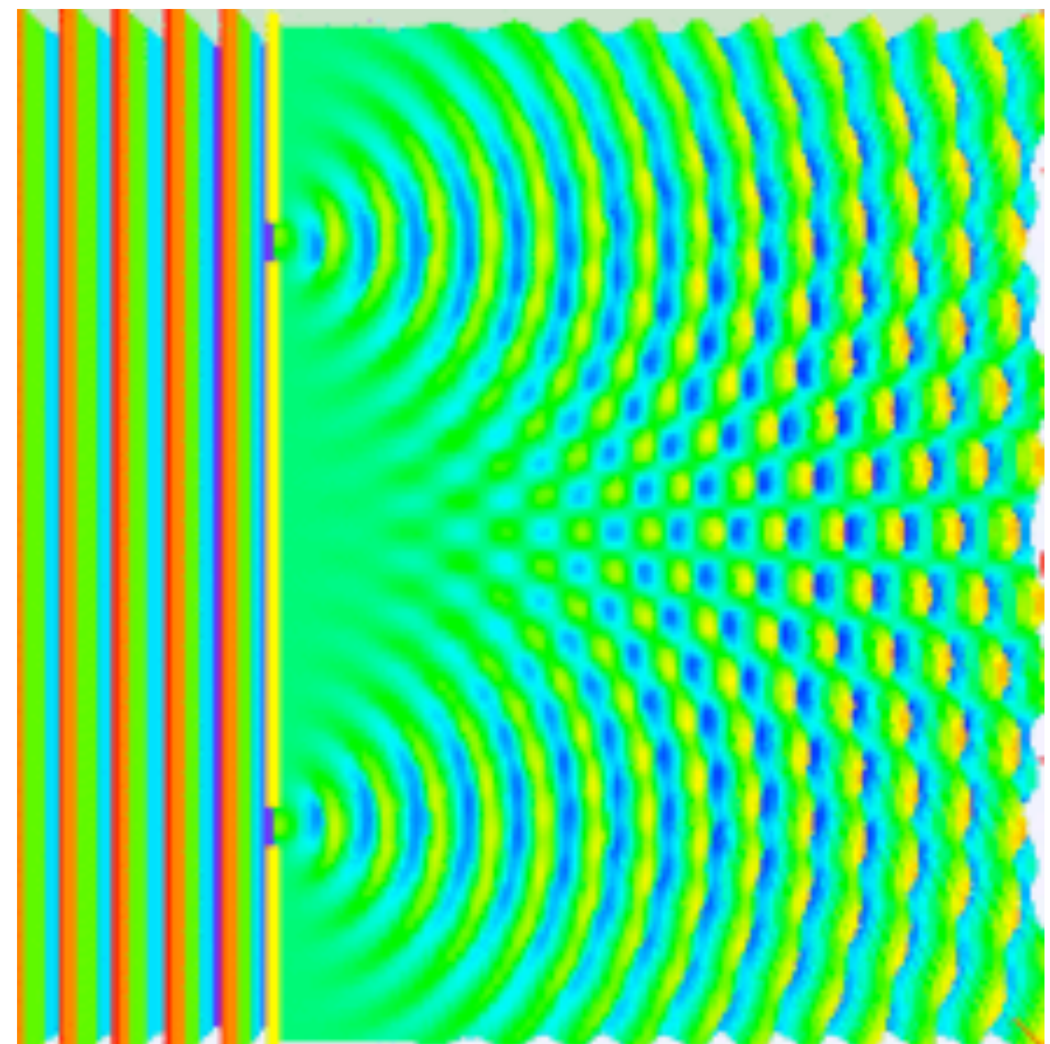
PHOTON SELF-IDENTITY PROBLEMS

Single-Slit and Double-Slit Diffraction

Diffraction = Light "bending" as it passes through an aperture or moves from one material to another.

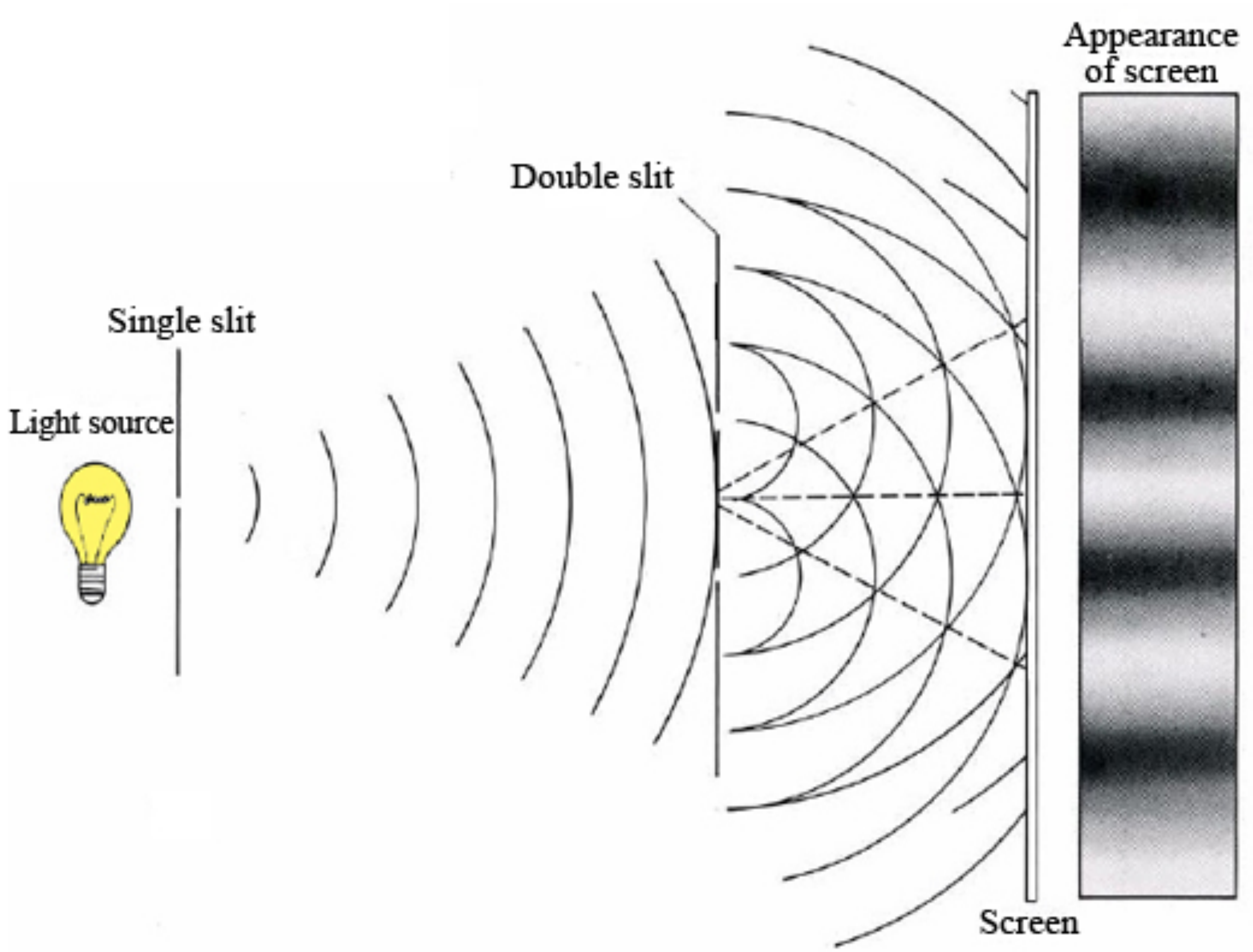


Single-Slit:
Single wavefront produced.



Double-Slit:
Two wavefronts overlap and interfere.

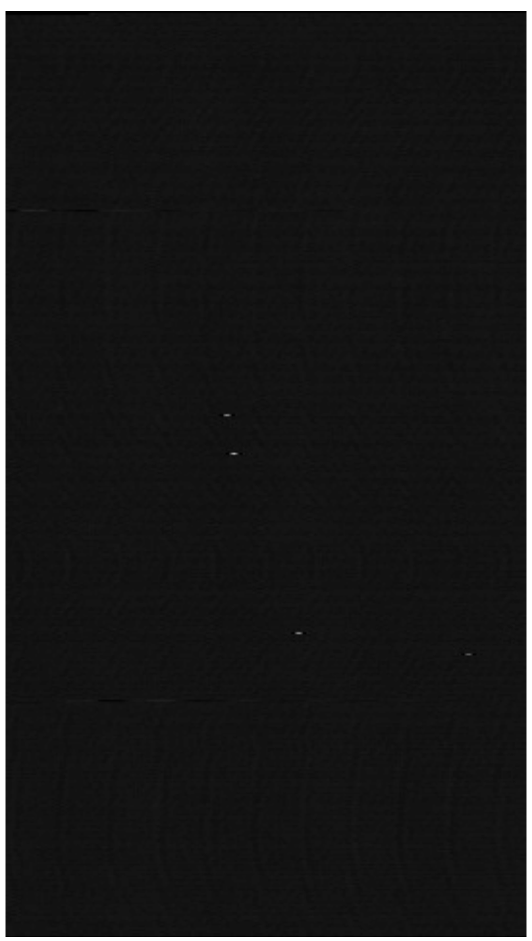
What do we see on a screen located behind the double-slit? A series of dark/bright "fringes", corresponding to constructive/destructive interference.



Question: If we decrease the intensity of the light source down to a single-photon (*i.e.* - there's never more than a single photon in the apparatus at a time), what should we see?

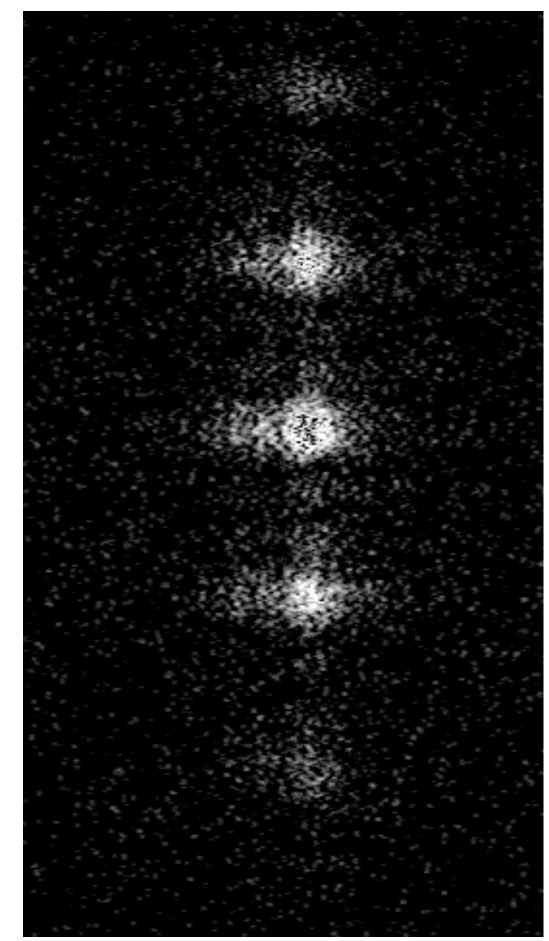
Logical Answer: If we decrease the intensity of the light source down to a single-photon, we should see two bright spots (one behind each slit), corresponding to the photon going through either the top or bottom slit.

But...



1/30 s

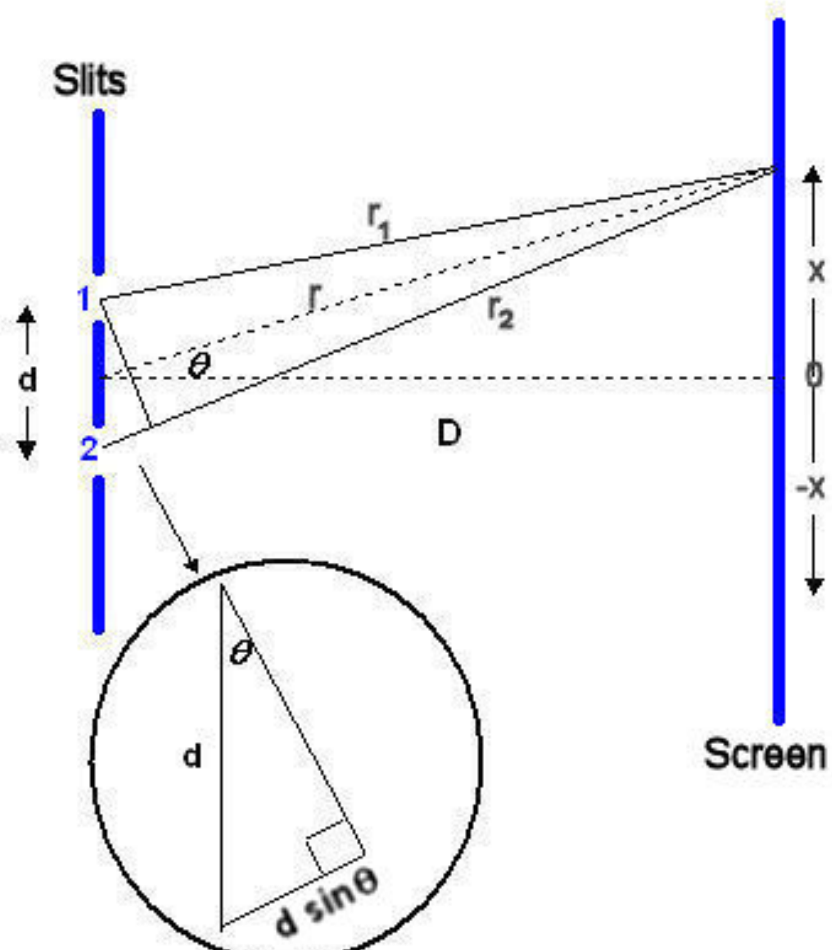
1 s



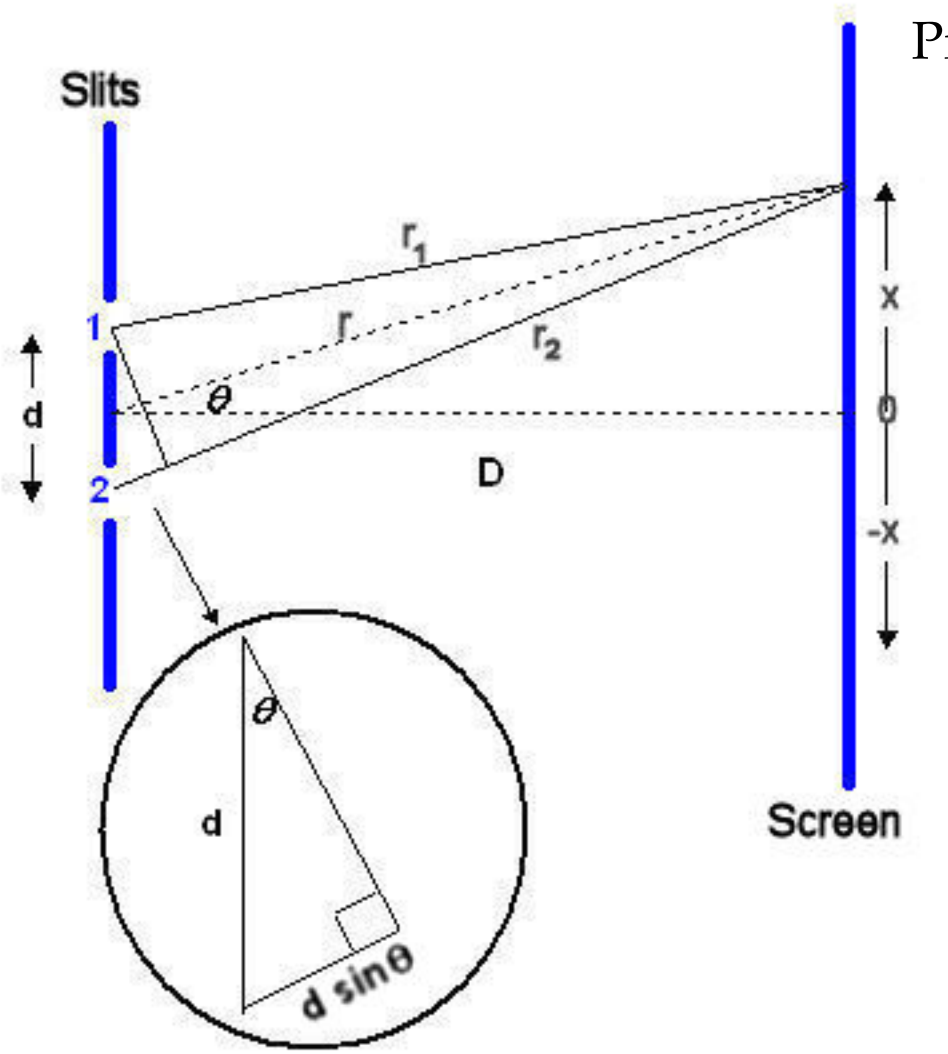
100 s

How do we explain this interference pattern?

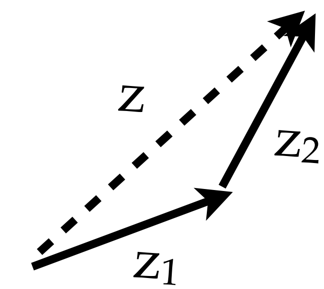
- Each photon has two possible paths to reach a given point on the screen (e.g. - through Slit 1 or 2).
- Quantum Mechanics says each path has an associated "**probability amplitude**", which is a complex number that describes that path.
- To determine the probability that a photon arrives at the point on the screen, we add the probability amplitudes and square.



We'll call the amplitudes for the two paths z_1 and z_2



Probability Amplitude $\mathbf{z} = \mathbf{z_1} + \mathbf{z_2}$



$$\text{Probability} = |\mathbf{z}|^2$$

$$= |\mathbf{z}_1|^2 + |\mathbf{z}_2|^2 + 2|\mathbf{z}_1||\mathbf{z}_2|\cos(\phi)$$

$$\text{if } \ |\mathbf{z}_1| = |\mathbf{z}_2|$$

$$= 2|\mathbf{z}_1|^2\,(1 + \cos(\phi))$$

$$\phi = \frac{2\pi \cdot d \cdot \sin\theta}{\lambda}$$

$$\text{Maximum Probability} = 4|\mathbf{z}_1|^2$$

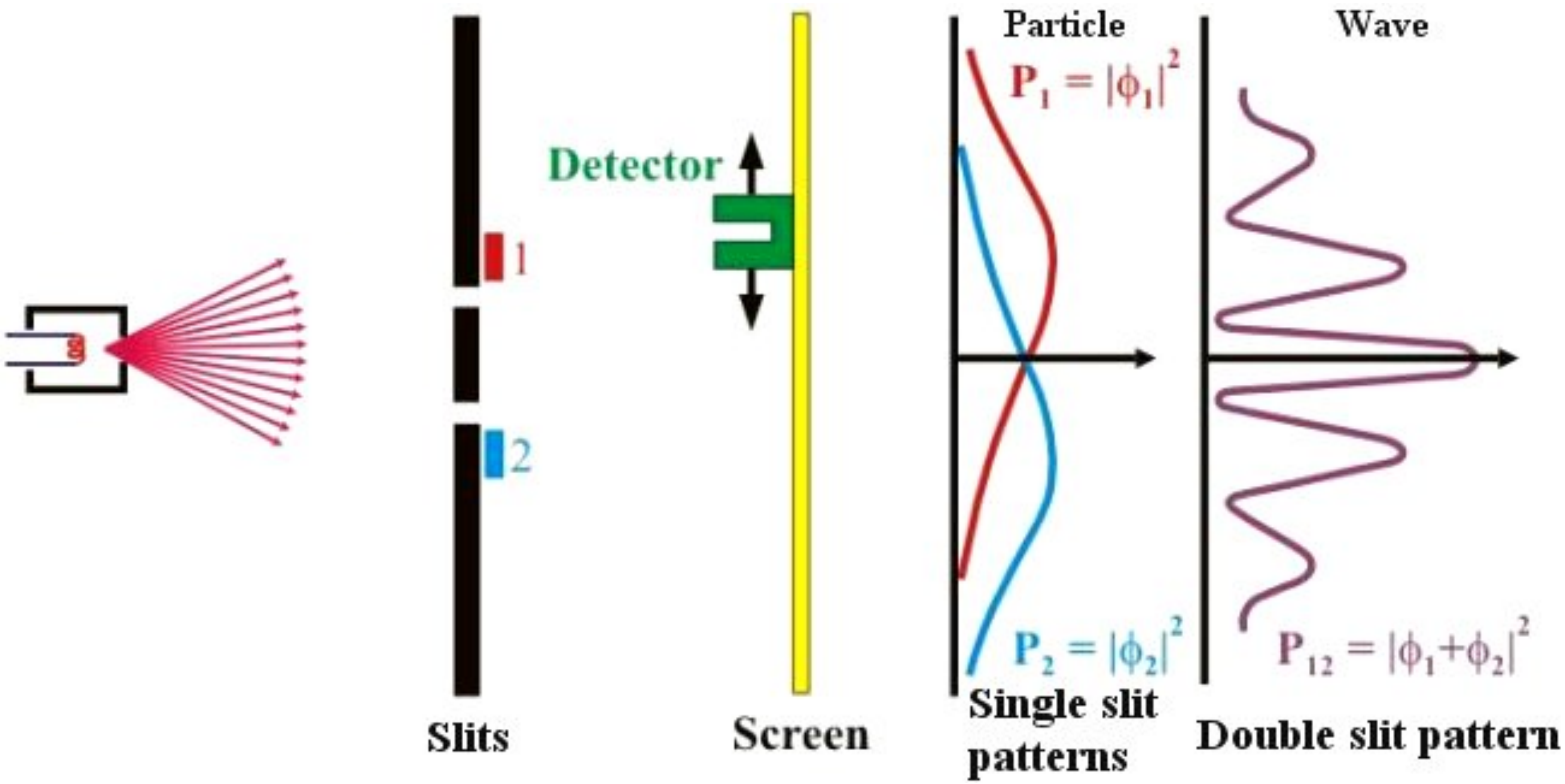
$$\phi = 2\pi \qquad d \cdot \sin\theta = \lambda$$

$$\text{Minimum Probability} = 0$$

$$\phi = \pi \qquad d \cdot \sin\theta = \frac{\lambda}{2}$$

The amplitudes intefere
with each other!

- If we close one of the slits, we get a peak behind the other slit.
- Even if we leave both slits open, but can somehow spy on one of the slits to know if it's the one the photon really went through, the interference pattern goes away!

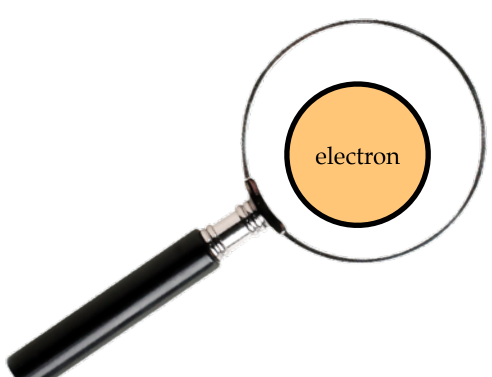


DeBroglie Wavelength

$$\lambda = \frac{h}{p}$$

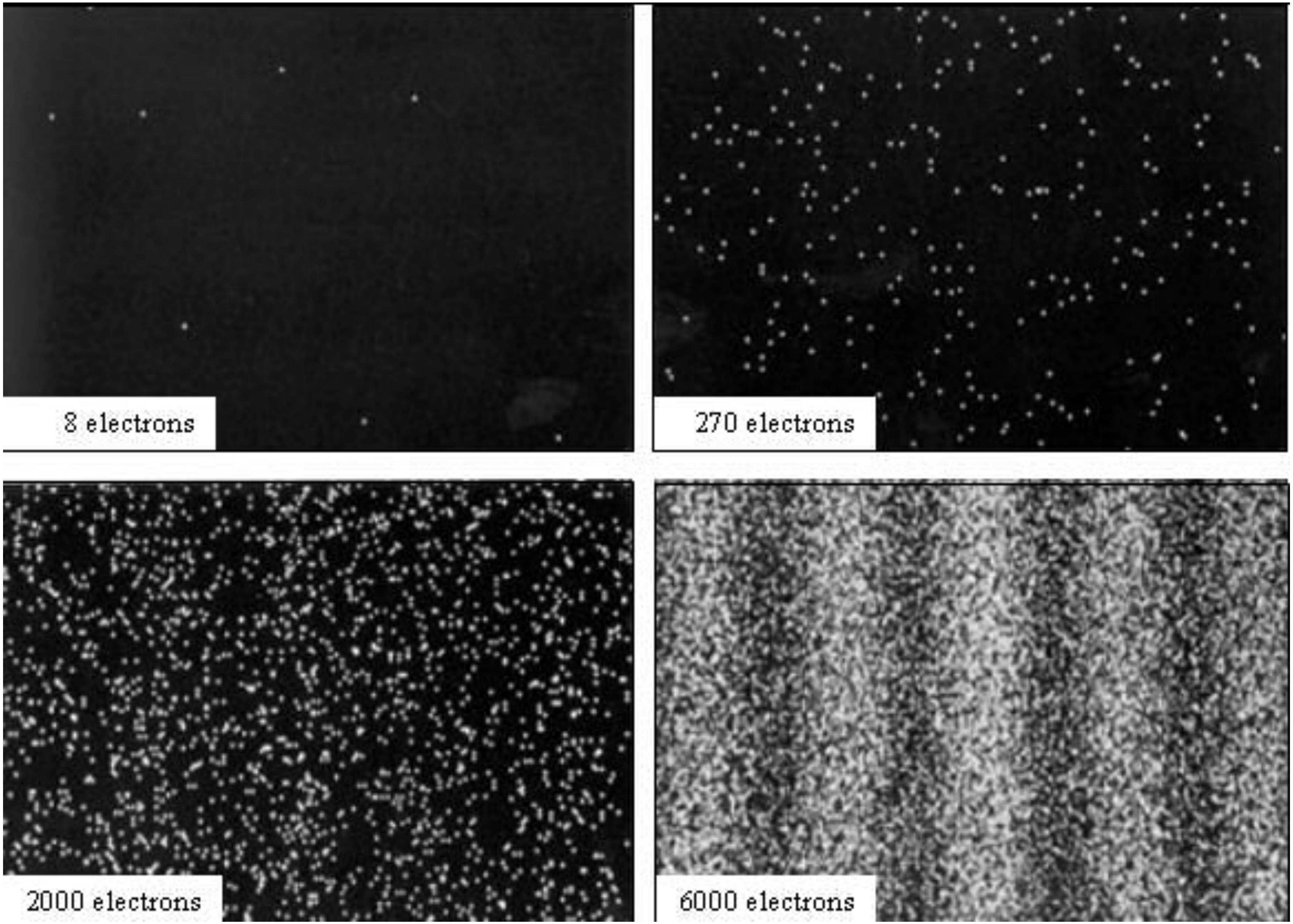
All objects, not just photons, have a wavelength!

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\mathrm{J \cdot s}}{0.1\mathrm{kg} \cdot 10\mathrm{m/s}} = 6.63 \times 10^{-34}\mathrm{m}$$



$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\mathrm{J \cdot s}}{9.1 \times 10^{-31}\mathrm{kg} \cdot 10\mathrm{m/s}} = 7.3 \times 10^{-5}\mathrm{m} = 7.3\mu m$$

Electrons in the Double-Slit Experiment



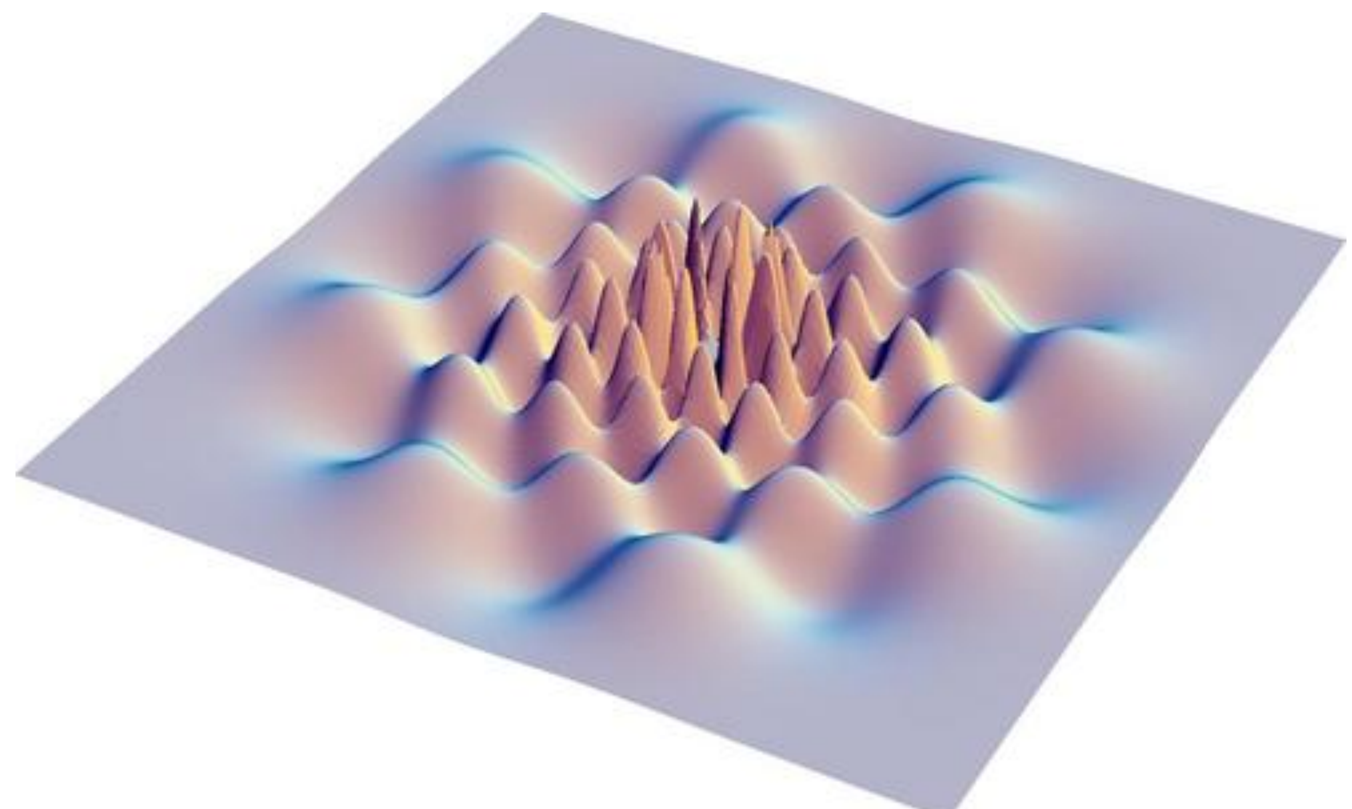
Interference pattern emerges, even though there is only ever one electron at a time in the experiment!

Quantum Mechanics:

The probability amplitude of an object(s) in a particular system (potential) is often called the **wavefunction**.

$$\Psi$$

Typical symbol for wavefunction



Determining the wavefunction is a primary task in Quantum Mechanics

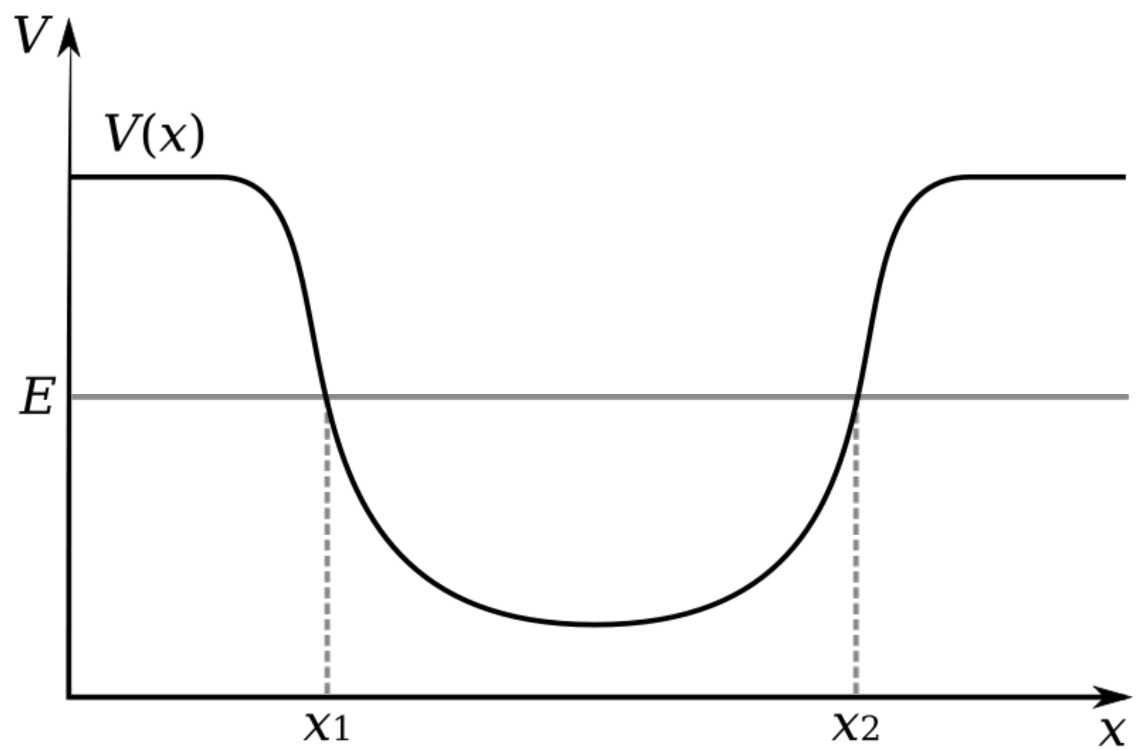
Schrodinger Equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \frac{-\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r},t)\Psi(\mathbf{r},t)$$

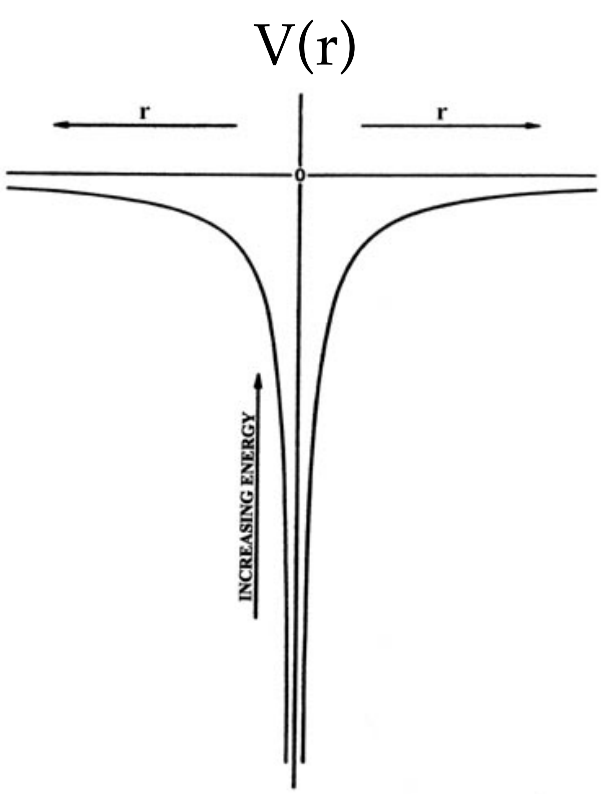
- This is the "F=ma" of Quantum Mechanics. It tells us how the wavefunction evolves in space and time.
- Physically allowed systems must have wavefunctions that solve the Schrodinger Equation.
- Determining the wavefunction lets us calculate physically meaningful quantities (Energies, momentum, position, etc...).

V(**r**,t) is the Potential Energy experienced by the quantum object at all points in space and time.

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \frac{-\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r},t)\Psi(\mathbf{r},t)$$

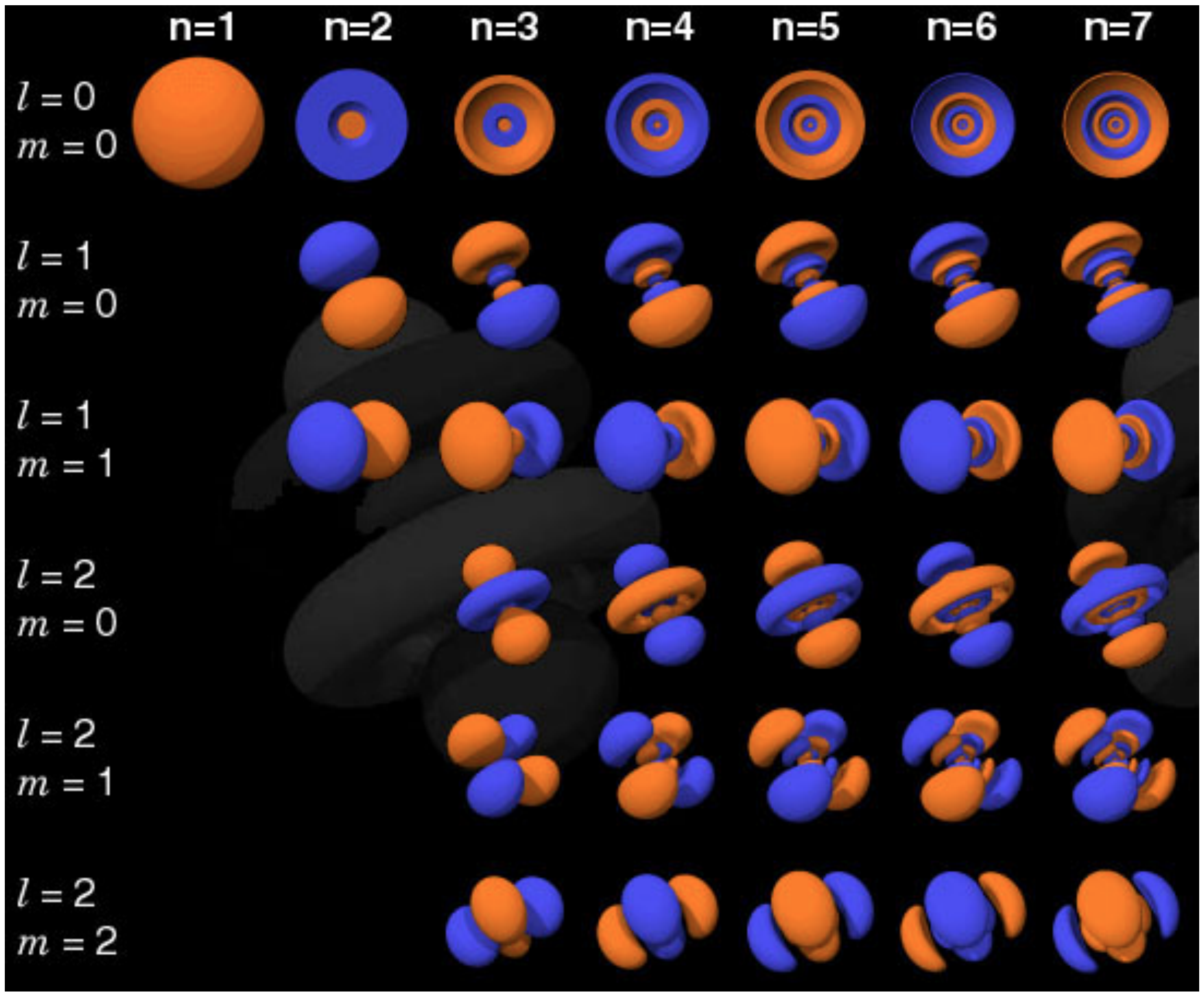


Arbitrary 1-D "Potential Well"



Potential an electron in a Hydrogen atom sees due to the Coulomb attraction of the Proton.

The Schrodinger Equation can be solved exactly for the Hydrogen Atom, explaining orbital shapes.



Hydrogen WaveFunction Shapes

$$E = -\frac{E_0}{n^2} \qquad E_0 = 13.6 \text{ eV}$$

HYDROGEN-ATOM WAVE FUNCTIONS, $n \leq 3$

$\rho \equiv \frac{r}{a_o}$

$n=1 \quad l=0 \quad m=0 \quad \Psi_{100} = \left(\frac{1}{\pi a_o^3}\right)^{1/2} e^{-\rho}$

$n=2 \quad l=0 \quad m=0 \quad \Psi_{200} = \frac{1}{8}\left(\frac{2}{\pi a_o^3}\right)^{1/2} (2-\rho)e^{-\rho/2}$

$n=2 \quad l=1 \quad m=0 \quad \Psi_{210} = \frac{1}{8}\left(\frac{2}{\pi a_o^3}\right)^{1/2} \rho e^{-\rho/2}\cos\theta$

$n=2 \quad l=1 \quad m=\pm1 \quad \Psi_{211} = \frac{1}{8}\left(\frac{1}{\pi a_o^3}\right)^{1/2} \rho e^{-\rho/2}\sin\theta\, e^{i\phi} \qquad \Psi_{21-1} = \frac{1}{8}\left(\frac{1}{\pi a_o^3}\right)^{1/2} \rho e^{-\rho/2}\sin\theta\, e^{-i\phi}$

$n=3 \quad l=0 \quad m=0 \quad \Psi_{300} = \frac{1}{243}\left(\frac{3}{\pi a_o^3}\right)^{1/2} (27 - 18\rho + 2\rho^2)e^{-\rho/3}$

$n=3 \quad l=1 \quad m=0 \quad \Psi_{310} = \frac{1}{81}\left(\frac{2}{\pi a_o^3}\right)^{1/2} \rho(6-\rho)e^{-\rho/3}\cos\theta$

$n=3 \quad l=1 \quad m=\pm1 \quad \Psi_{311} = \frac{1}{81}\left(\frac{1}{\pi a_o^3}\right)^{1/2} \rho(6-\rho)e^{-\rho/3}\sin\theta\, e^{i\phi} \qquad \Psi_{31-1} = \frac{1}{81}\left(\frac{1}{\pi a_o^3}\right)^{1/2} \rho(6-\rho)e^{-\rho/3}\sin\theta\, e^{-i\phi}$

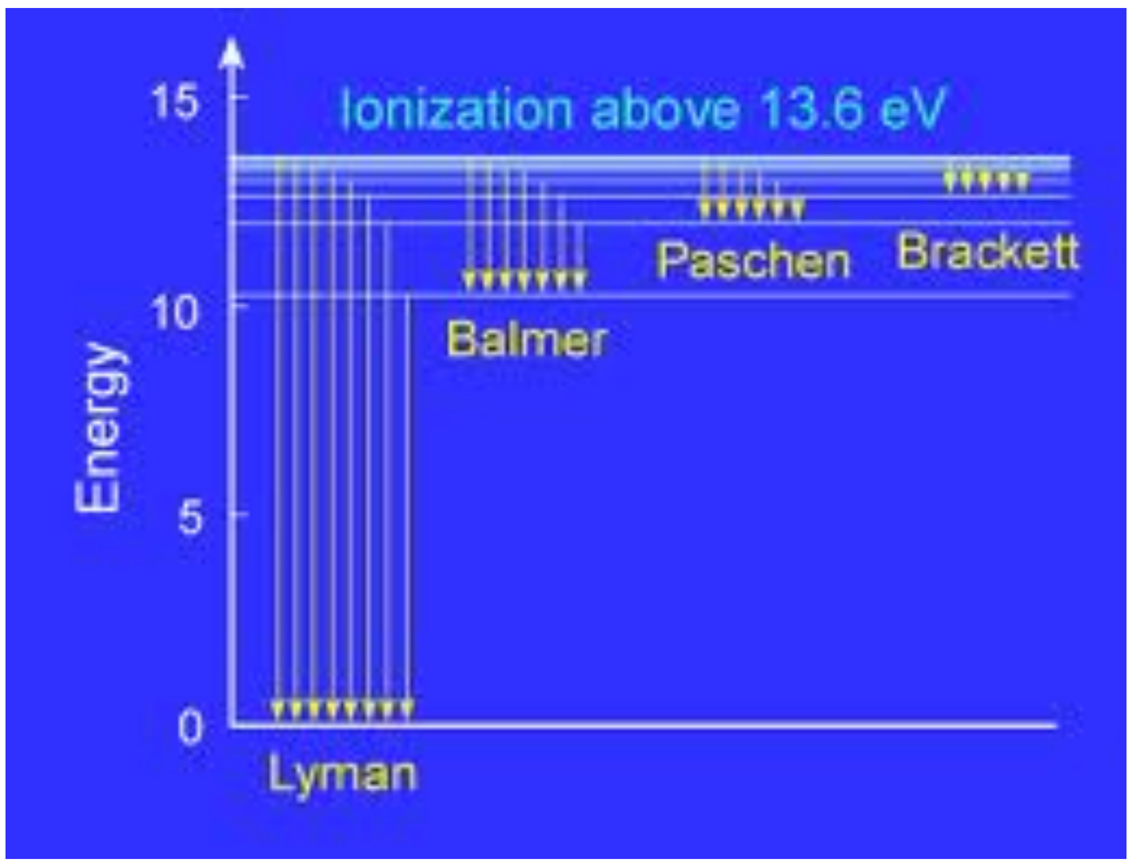
$n=3 \quad l=2 \quad m=0 \quad \Psi_{320} = \frac{1}{486}\left(\frac{6}{\pi a_o^3}\right)^{1/2} \rho^2 e^{-\rho/3}(3\cos^2\theta - 1)$

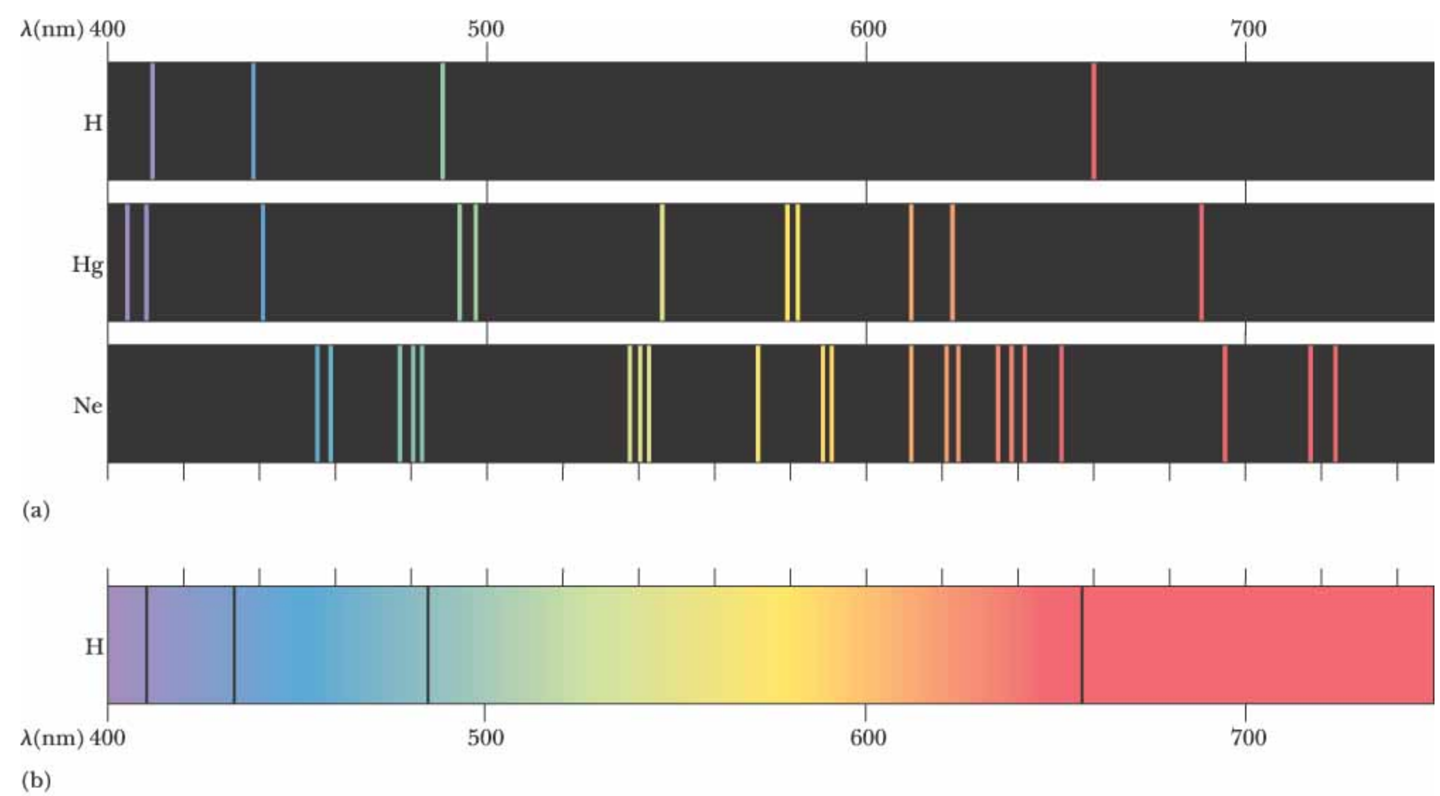
$n=3 \quad l=2 \quad m=\pm1 \quad \Psi_{321} = \frac{1}{81}\left(\frac{1}{\pi a_o^3}\right)^{1/2} \rho^2 e^{-\rho/3}\sin\theta\cos\theta\, e^{i\phi} \qquad \Psi_{32-1} = \frac{1}{81}\left(\frac{1}{\pi a_o^3}\right)^{1/2} \rho^2 e^{-\rho/3}\sin\theta\cos\theta\, e^{-i\phi}$

$n=3 \quad l=2 \quad m=\pm2 \quad \Psi_{322} = \frac{1}{162}\left(\frac{1}{\pi a_o^3}\right)^{1/2} \rho^2 e^{-\rho/3}\sin^2\theta\, e^{i2\phi} \qquad \Psi_{32-2} = \frac{1}{162}\left(\frac{1}{\pi a_o^3}\right)^{1/2} \rho^2 e^{-\rho/3}\sin^2\theta\, e^{-i2\phi}$

Quantum Numbers

- Solution to Schrodinger Equation for Hydrogen also explains emission spectrum.
- Photons are emitted when the electron in an atom drops from a higher energy level to a lower one.

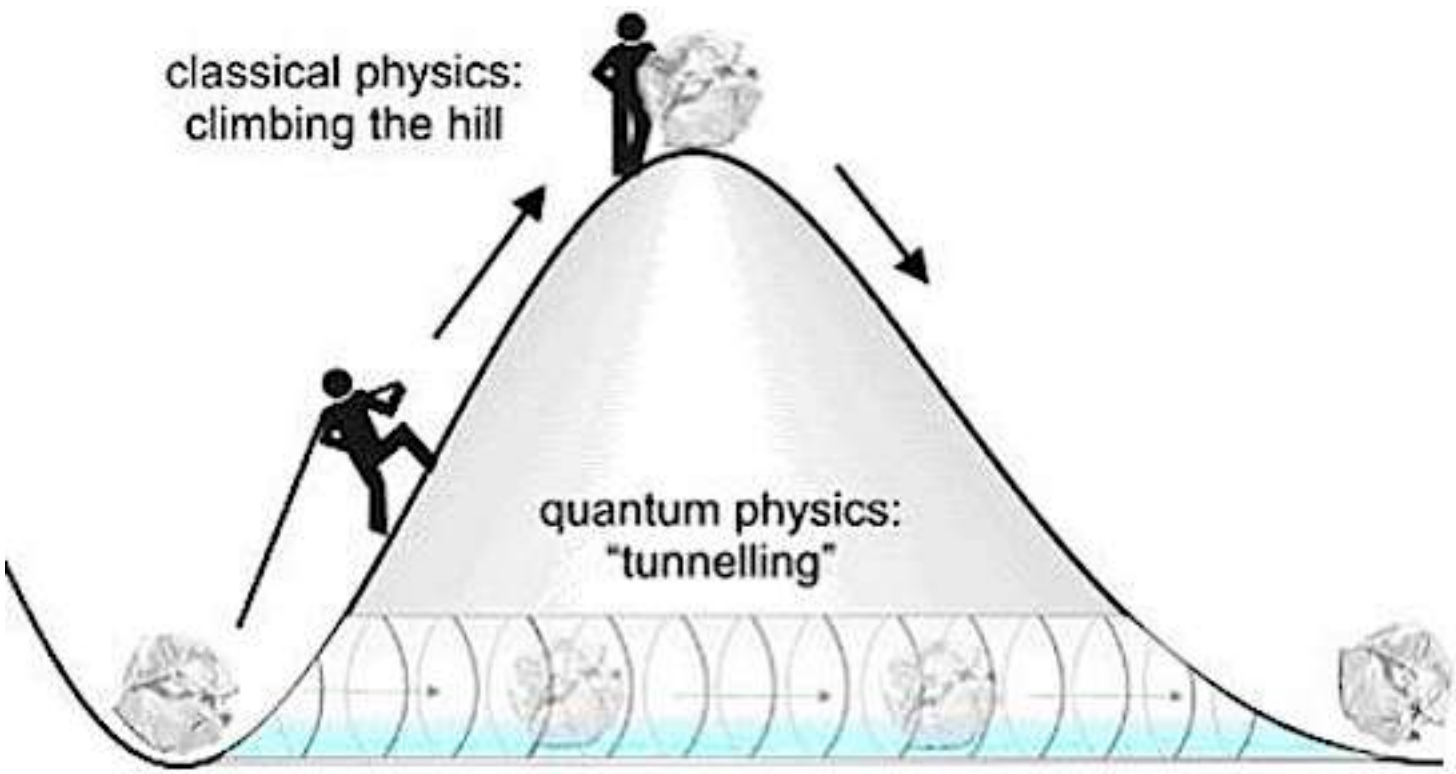


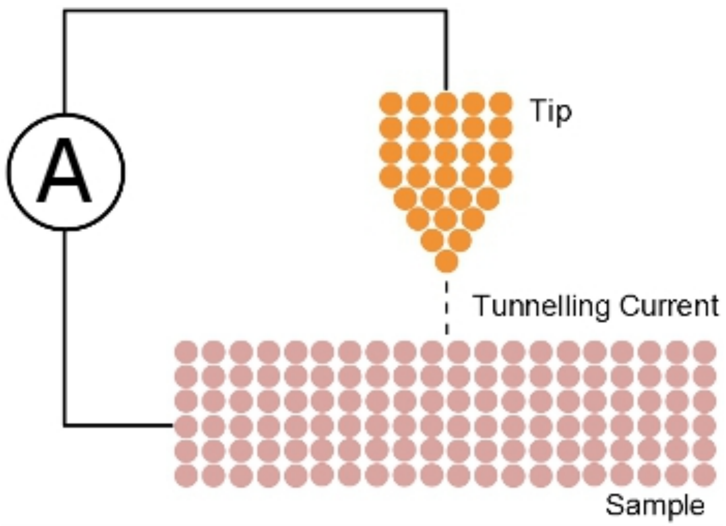


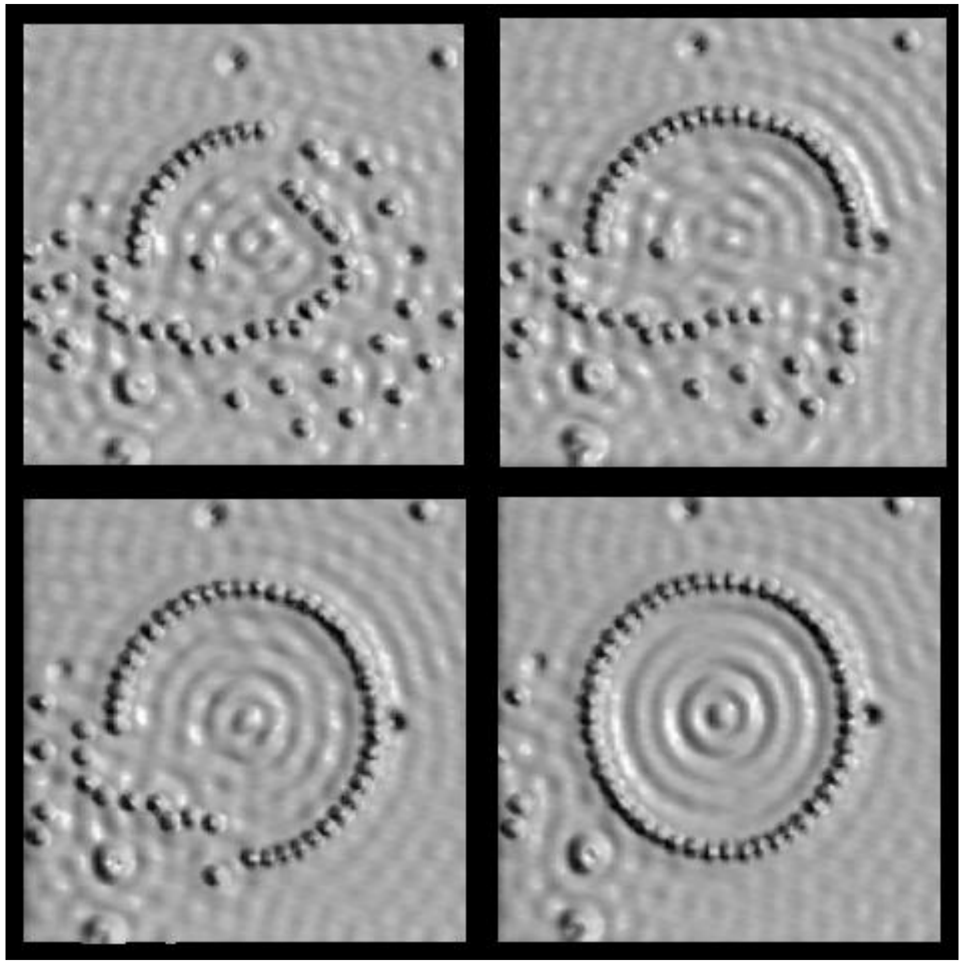
$$E_{\text{photon}} = -E_0\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

$$E_{\text{photon}} = h\nu$$

Quantum objects can "tunnel" through barriers that classically they should bounce off of.



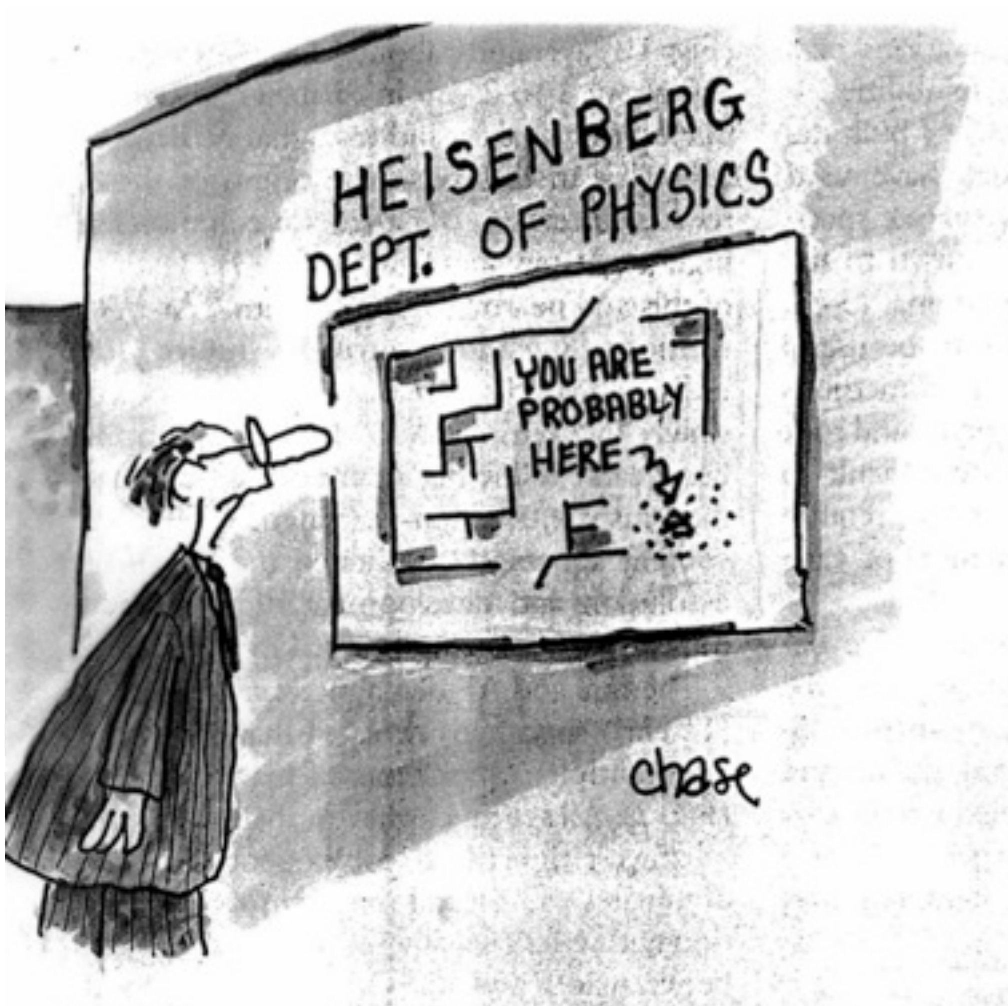




48 Iron atoms on Copper

Heisenberg Uncertainty Principle

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

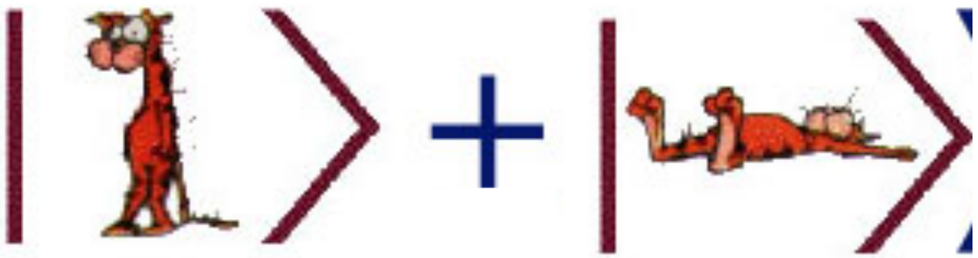


You cannot simultaneously know an objects position and momentum with infinite precision.

also... $\Delta E \Delta t \geq \frac{\hbar}{2}$

Schrodinger's Cat

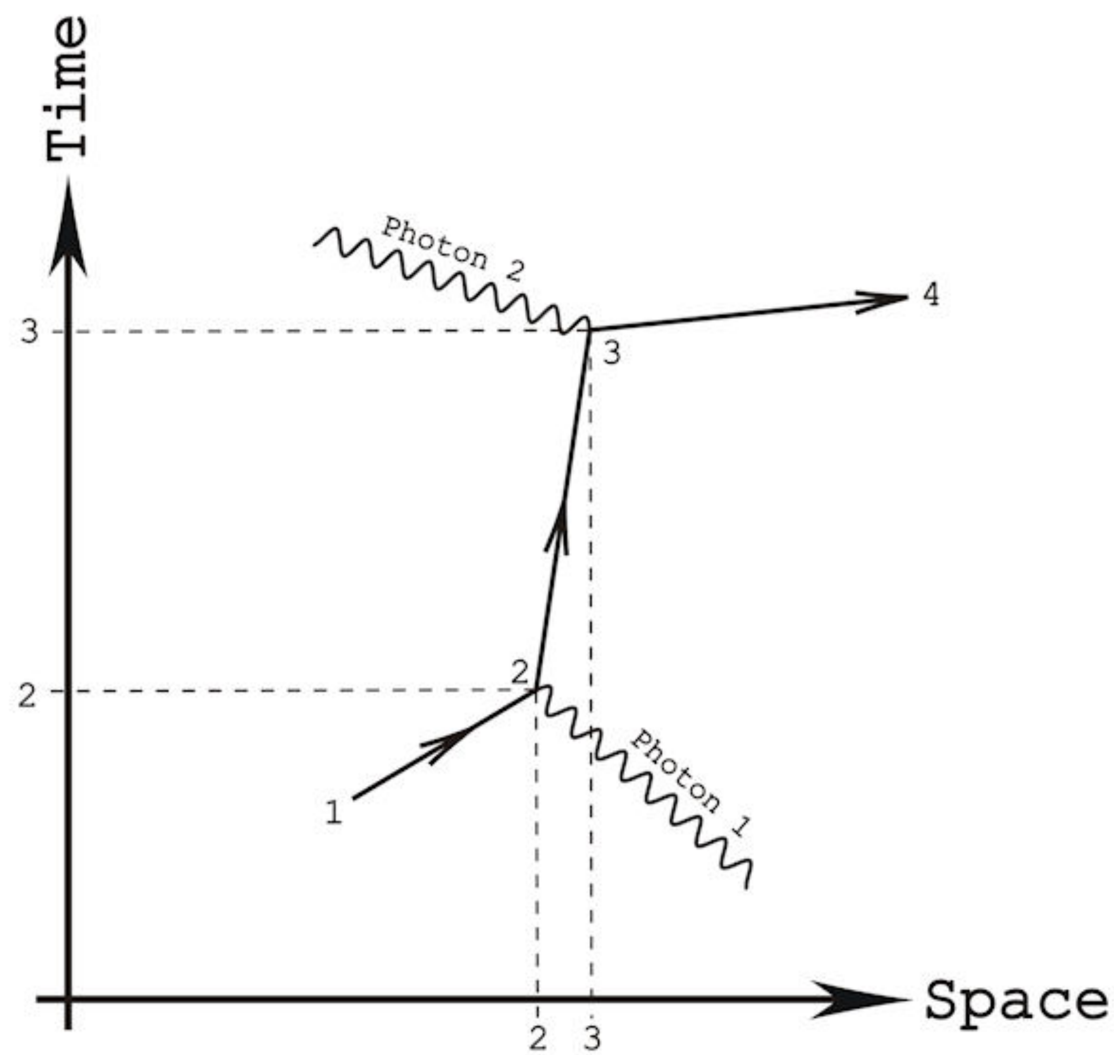
Does observation changes a quantum system?



"One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following device (which must be secured against direct interference by the cat): in a Geiger counter there is a tiny bit of radioactive substance, so small, that perhaps in the course of the hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives if meanwhile no atom has decayed. The psi-function of the entire system would express this by having in it the living and dead cat (pardon the expression) mixed or smeared out in equal parts."—Erwin Schrödinger

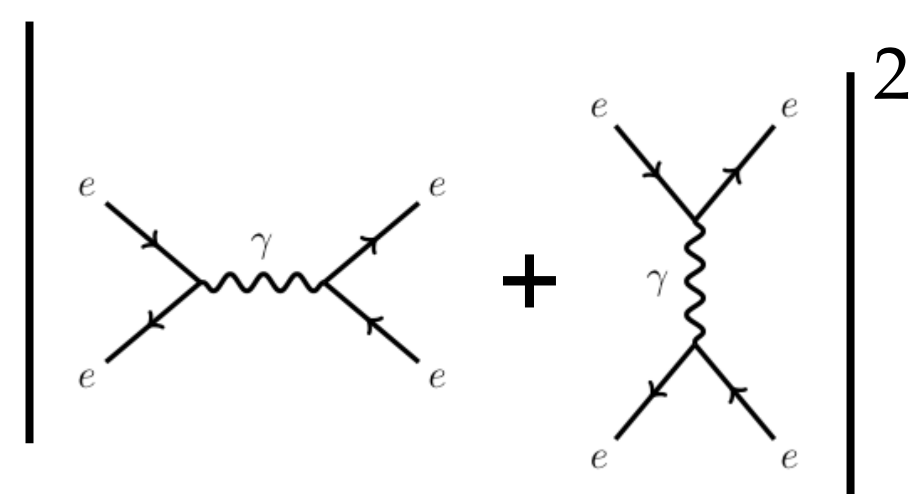
Quantum Mechanics and Particle Physics

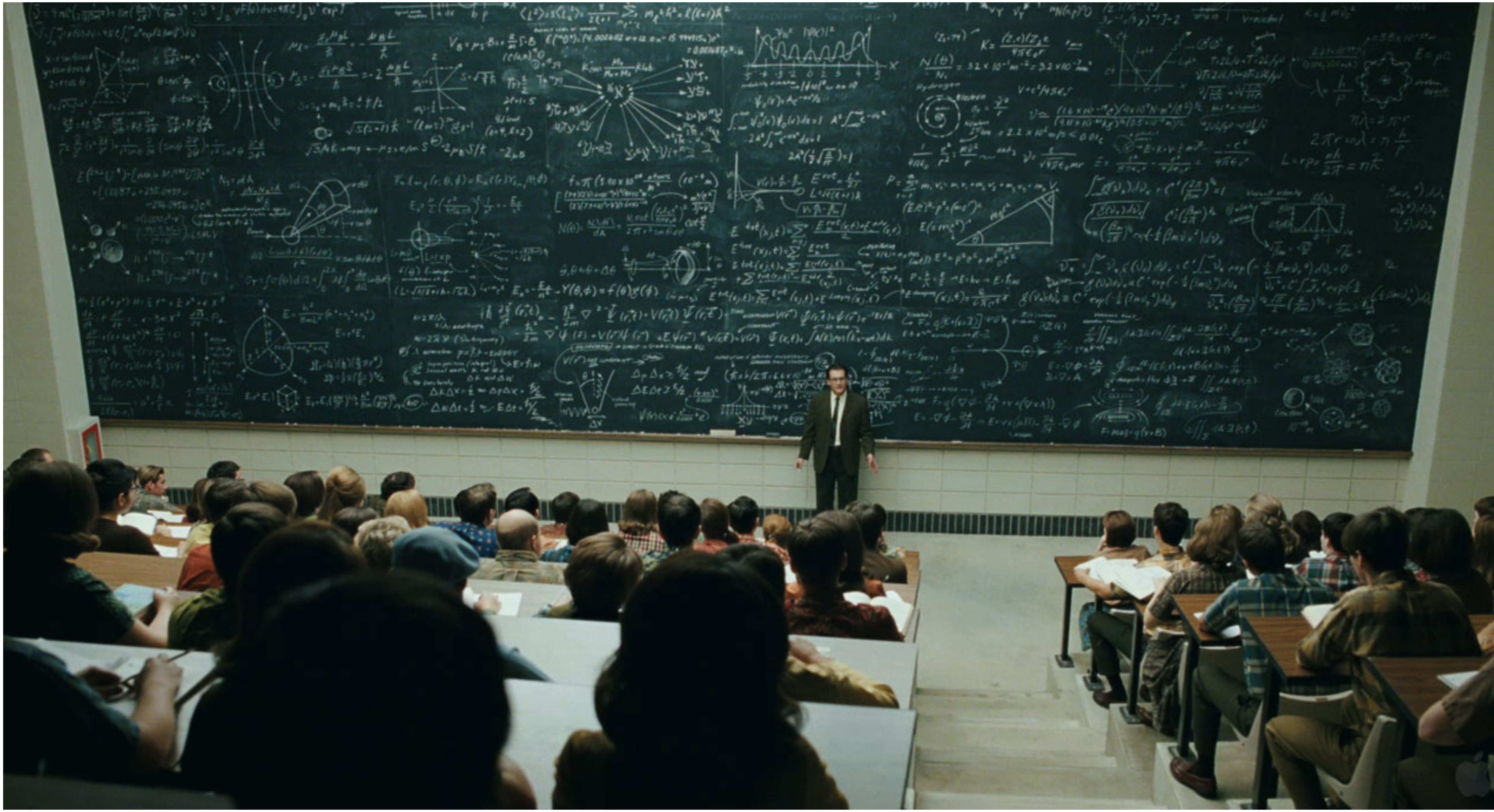
- The theoretical language of particle physics is built on quantum mechanics (relativistic quantum mechanics to be precise)
- Particle interactions have probability amplitudes associated with them (Feynman diagrams are the graphical representation), that are used to calculate the likelihood of the interactions taking place.
- If a process can happen multiple ways, have to add probability amplitudes for each way.



Feynman Diagrams

Probability ($e^+e^- \to e^+e^-$) =





Questions?

Schrodinger Equation

Heisenberg Uncertainty Relations